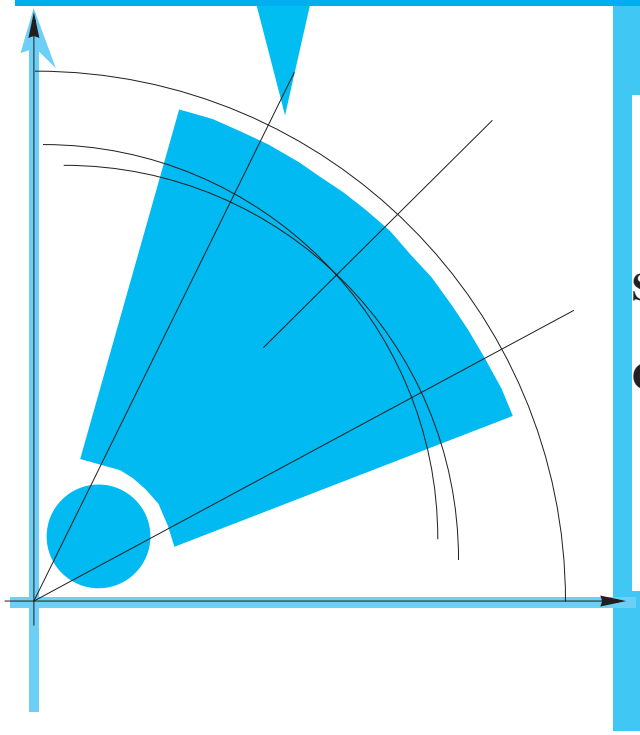
INTERNATIONAL BACCALAUREATE

ANSWERS

MATHEMATICS STANDARD LEVEL



3RD EDITION
3rd Imprint

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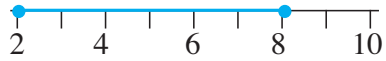
Contributing author
Patrick Tobin

EXERCISE 2.1

1. (a)


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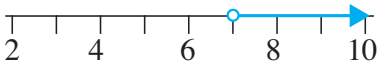
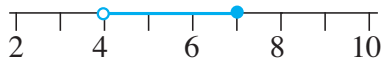


(c)


(d)


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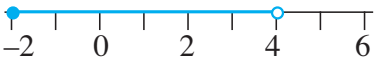
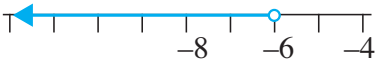
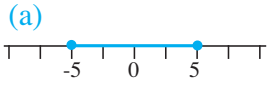


(f)


2. (a) $[-2,7]$ (b) $]9,\infty[$ (c) $]0,5]$ (d) $]-\infty,0]$ (e) $]-4,8[$ (f) $]-\infty,-1[\cup]2,\infty[$ **3.** (a) $5\sqrt{5}$ (b) $-\sqrt{3}$ (c) $\sqrt{3}$ **4.** (a) 4 (b) $4+\sqrt{6}$ (c) $6\sqrt{2}$ (d) $31+12\sqrt{3}$ **5.** (a) $2-\sqrt{3}$ (b) $\sqrt{7}+2$ (c) $2\sqrt{3}+\sqrt{15}$ (d) $-2-\sqrt{3}-4\sqrt{5}-2\sqrt{15}$ (e) $\dfrac{3+\sqrt{6}+\sqrt{10}+\sqrt{15}}{-2}$ (f) $3\sqrt{6}+2\sqrt{15}$ **6.** (a) i. $\dfrac{3\sqrt{5}+\sqrt{3}}{2}$ ii. $10+\dfrac{3\sqrt{15}}{2}$ (b) i. $\dfrac{14\sqrt{3}+48}{13}$ ii. $\dfrac{1344\sqrt{3}+3230}{169}$ **7.** (a) $\{\pm 3\}$ (b) $\{\pm 10\}$ (c) $\varnothing$ (d) $\{-4,2\}$ (e) $\{-12,8\}$ (f) $\{0,4\}$

8. (a)


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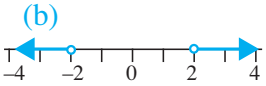


(c)


(d)


9. (a) $]1,\infty[$ (b) $]4,\infty[$ (c) $]4,6[$

EXERCISE 2.2.1

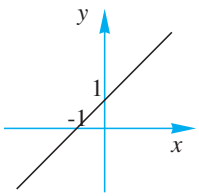
1. (a) 4 (b) 3 (c) –6 (d) $-\dfrac{11}{2}$ (e) $\dfrac{1}{10}$ (f) $\dfrac{3}{8}$ **2.** (a) $\dfrac{17}{5}$ (b) $\dfrac{4}{3}$ (c) $-\dfrac{3}{4}$ (d) $\dfrac{4}{3}$ (e) $\dfrac{35}{2}$ (f) $\dfrac{92}{41}$

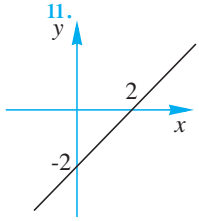
3. (a) $-\dfrac{44}{5}$ (b) –39 (c) $-\dfrac{1}{7}$ (d) –3 (e) 2 (f) 4 **4.** (a) $2b-2$ (b) $b+1+\dfrac{b}{a}$ (c) $\dfrac{ab}{a+b}$ (d) $a(a+b)$ (e) ab (f) $\dfrac{ab}{a-b}$ (g) 0 (h) $\dfrac{a+b}{a^2+b^2}$ (i) $a+b$ **5.** (a) $-4, 4$ (b) $-\dfrac{9}{5}, 3$ (c) $-6, 18$ (d) $-\dfrac{11}{2}, \dfrac{17}{2}$ (e) $-\dfrac{7}{10}, \dfrac{1}{10}$ (f) $-\dfrac{5}{8}, \dfrac{3}{8}$ (g) $-\dfrac{7}{5}, \dfrac{17}{5}$ (h) $\dfrac{4}{3}, \dfrac{20}{3}$ (i) $-3, 0$ (j) $\dfrac{a-b}{2}, \dfrac{b-a}{2}, a \geq b$ (k) $\pm b(a-b), a \geq b$ (l) $-\dfrac{b}{a}, \dfrac{2b}{a}, b \geq 0$

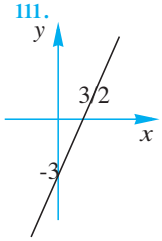
EXERCISE 2.2.2

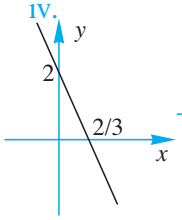
1. (a) $x < -4$ (b) $x \leq -\dfrac{1}{5}$ (c) $x > 1$ (d) $x \leq -6$ (e) $x > \dfrac{18}{7}$ (f) $x > \dfrac{3}{8}$ **2.** (a) $x > \dfrac{52}{11}$ (b) $x \leq 1$ (c) $x \leq \dfrac{10}{3}$ **3.** (a) $x < 1$ (b) $x < 2-a$ (c) $x > \dfrac{2b}{3a}$ (d) $x \geq \dfrac{2}{(a+1)^2}$ **4.** (a) $-2 \leq x \leq 1$ (b) $-2 \leq x \leq 3$ (c) $-\dfrac{3}{2} \leq x \leq \dfrac{5}{2}$ (d) $x = -\dfrac{1}{2}$ (e) $-7 \leq x \leq 9$ (f) $-5 \leq x \leq 3$ (g) $-4 \leq x \leq 16$ (h) $-28 \leq x \leq 44$ (i) $-\dfrac{5}{12} \leq x \leq \dfrac{1}{12}$ **5.** (a) $x < -\dfrac{3}{2} \cup x > \dfrac{5}{2}$ (b) $x < \dfrac{3}{2} \cup x > \dfrac{7}{2}$ (c) $x \leq -12 \cup x \geq 16$ (d) $x \leq -24 \cup x \geq 6$ (e) $x < \dfrac{3}{4} \cup x > \dfrac{9}{4}$ (f) $-6 < x < 14$ (g) $x < -28 \cup x > 44$ (h) $x < -\dfrac{5}{12} \cup x > \dfrac{1}{12}$ (i) $x \leq -4 \cup x \geq 16$ **6.** $p < 3$

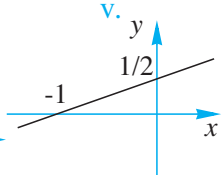
EXERCISE 2.3.1

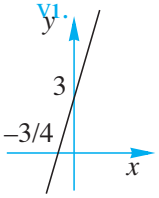
1. i. 

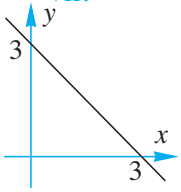
ii. 

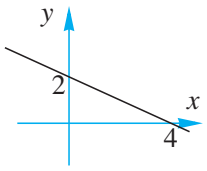
iii. 

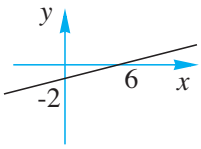
iv. 

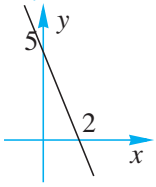
v. 

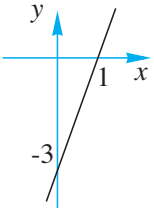
vi. 

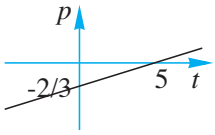
vii. 

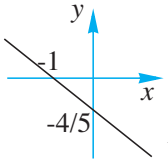
viii. 

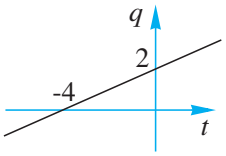
ix. 

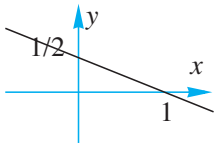
x. 

xi. 

xii. 

xiii. 

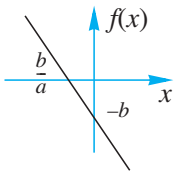
xiv. 

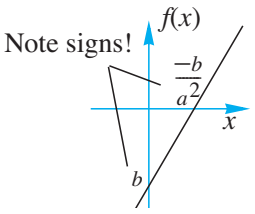
xv. 

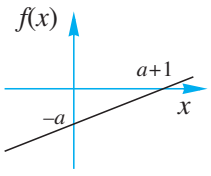
2. (a) 2 (b) 3 (c) $\frac{5}{3}$ **3.** (a) $y = 2x - 1$ (b) $y = 3x + 9$ (c) $y = -x - 1$ **4.** (a) $-\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{3}{2}$

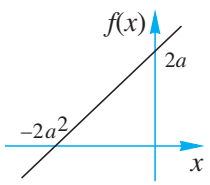
(d) $-\frac{4}{5}$ **5.** $y = 2x$ **6.** $y = -x + 1$ **7.** $y = \frac{x+2}{2}$ **8.** 2 **9.** (a) $y = \frac{5}{2}x$ (b) $y = -\frac{3}{2}x + 3$

(c) $y = \frac{5}{6}x - \frac{1}{2}$ (d) $y = -2x + 1$

10. (a) 

(b) 

(c) 

(d) 

EXERCISE 2.3.2

1. (i) $x = 1, y = 2$ (ii) $x = 3, y = 5$ (iii) $x = -1, y = 2$ (iv) $x = 0, y = 1$ (v) $x = -2, y = -3$

(vi) $x = -5, y = 1$ **2.** (i) $x = \frac{13}{11}, y = \frac{17}{11}$ (ii) $x = \frac{9}{14}, y = \frac{3}{14}$ (iii) $x = 0, y = 0$

(iv) $x = \frac{4}{17}, y = -\frac{22}{17}$ (v) $x = -\frac{16}{7}, y = \frac{78}{7}$ (vi) $x = \frac{5}{42}, y = -\frac{3}{28}$ **3.** (i) –3 (ii) –5 (iii) –1.5

4. (i) $m = 2, a = 8$ (ii) $m = 10, a = 24$ (iii) $m = -6, a = 9$.

5. (a) $x = 1, y = a - b$ (b) $x = -1, y = a + b$ (c) $x = \frac{1}{a}, y = 0$ (d) $x = b, y = 0$

(e) $x = \frac{a-b}{a+b}, y = \frac{a-b}{a+b}$ (f) $x = a, y = b - a^2$

EXERCISE 2.3.3

1. (a) $x = 4, y = -5, z = 1$ (b) $x = 0, y = 4, z = -2$ (c) $x = 10, y = -7, z = 2$

(d) $x = 1, y = 2, z = -2$ (e) $\varnothing$ (f) $x = 2t - 1, y = t, z = t, t \in \mathbb{R}$

EXERCISE 2.4.1

1. (a) –5 (b) 4, 6 (c) –3, 0 (d) 1, 3 (e) –6, 3 (f) –2, 5/3 (g) 2 (h) –3, 6 (i) –6, 1 (j) 0, 1.5

2. (a) –1 (b) –7, 5 (c) –0.4, 3 (d) –2, 1 (e) –3, 1 (f) 4, 5

3. (a) $-1 \pm \sqrt{6}$ (b) $3 \pm \sqrt{5}$ (c) $1 \pm \sqrt{5}$ (d) $\frac{-1 \pm \sqrt{33}}{8}$ (e) $\frac{9 \pm \sqrt{73}}{4}$ (f) $\frac{1 \pm \sqrt{85}}{6}$

4. (a) $\frac{3 \pm \sqrt{37}}{2}$ (b) $\frac{5 \pm \sqrt{33}}{2}$ (c) $\frac{3 \pm \sqrt{33}}{2}$ (d) $\frac{7 \pm \sqrt{57}}{2}$ (e) $\frac{-7 \pm \sqrt{65}}{2}$ (f) –4, 2 (g) $-1 \pm 2\sqrt{2}$

(h) $\frac{-5 \pm \sqrt{53}}{2}$ (i) $\frac{3 \pm \sqrt{37}}{2}$ (j) no real solutions (k) $4 \pm \sqrt{7}$ (l) no real solutions (m) $\frac{2 \pm \sqrt{13}}{2}$

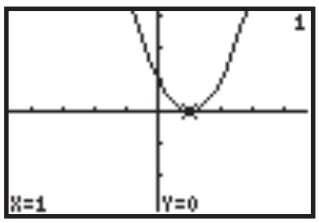
(n) $\frac{3 \pm 2\sqrt{11}}{5}$ (o) $\frac{6 \pm \sqrt{31}}{5}$ **5.** (a) $-2 < p < 2$ (b) $p = \pm 2$ (c) $p < -2$ or $p > 2$

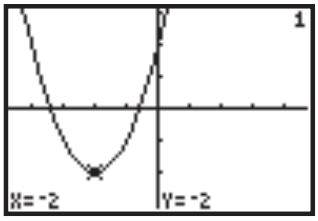
6. (a) $m = 1$ (b) $m < 1$ (c) $m > 1$ **7.** (a) $m = \pm 2\sqrt{2}$ (b) $]-\infty, -2\sqrt{2}[\cup]2\sqrt{2}, \infty[$

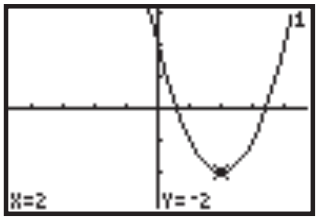
(c) $]-2\sqrt{2}, 2\sqrt{2}[$ **8.** (a) $k = \pm 6\sqrt{2}$ (b) $]-\infty, -6\sqrt{2}[\cup]6\sqrt{2}, \infty[$ (c) $]-6\sqrt{2}, 6\sqrt{2}[$

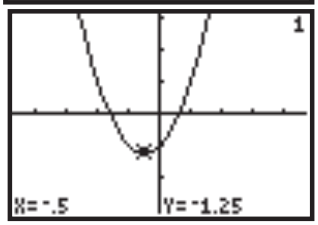
EXERCISE 2.4.2

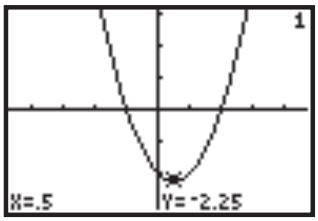
1. Graphs are shown using the ZOOM4 viewing window:

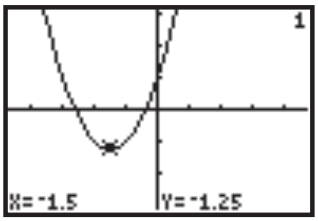
(a) 

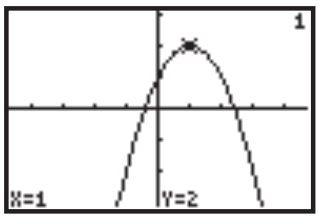
(b) 

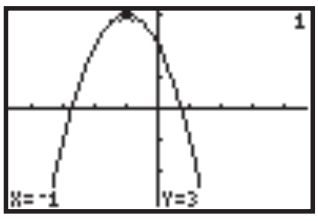
(c) 

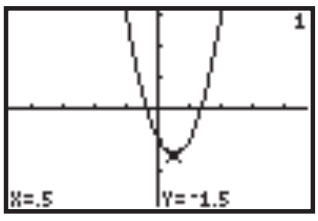
(d) 

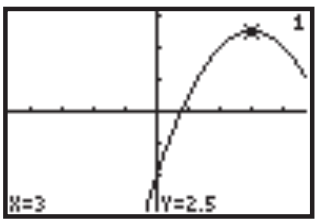
(e) 

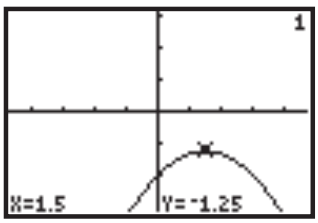
(f) 

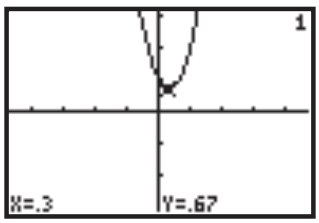
(g) 

(h) 

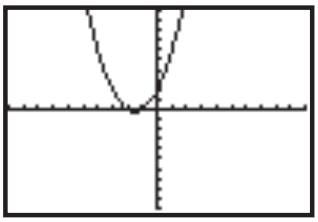
(i) 

(j) 

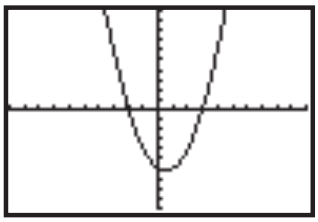
(k) 

(l) 

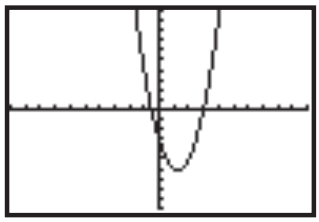
2. Graphs are shown using the ZOOM6 viewing window:

(a) 

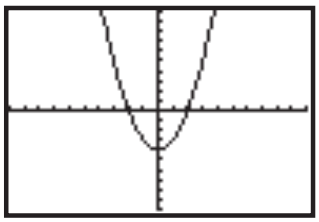
(–2,0),(–1,0),(0,2)

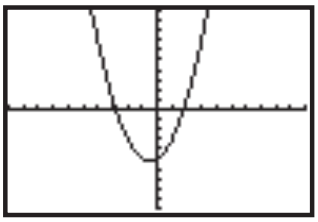
(b) 

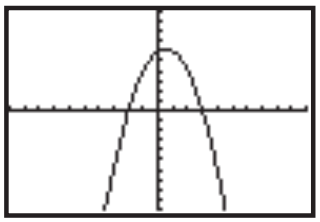
(–2,0),(3,0),(0,–6)

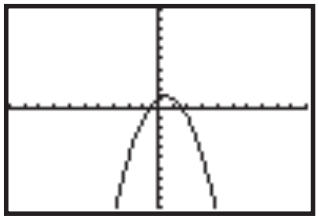
(c) 

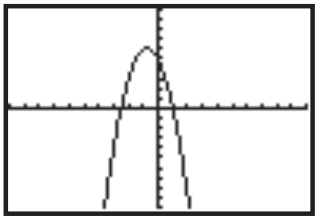
(–0.5,0),(3,0),(0,–3)

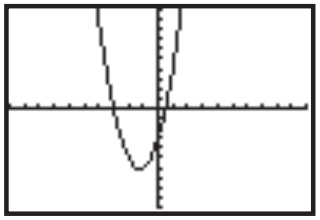
(d) 
(–2,0),(2,0),(0,–4)

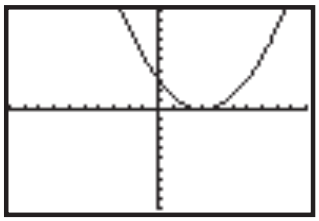
(e) 
(–2.79,0),(1.79,0),(0,–5)

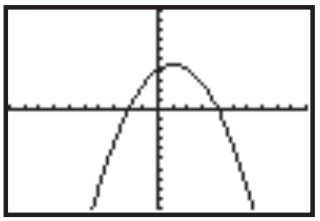
(f) 
(–2,0),(3,0),(0,6)

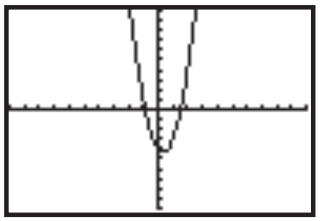
(g) 
(–0.62,0),(1.62,0),(0,1)

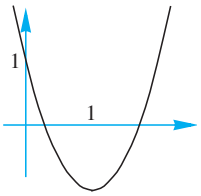
(h) 
(–2.5,0),(1,0),(0,5)

(i) 
(–3,0),(0.5,0),(0,–3)

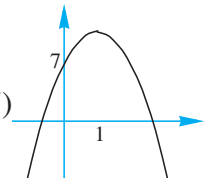
(j) 
(3,0),(0,3)

(k) 
(–2,0),(4,0),(0,4)ß

(l) 
(–0.87,0),(1.54,0),(0,–4)

3. (a) $x = 1$ (b) (1,–1) (c) i. $\left(\frac{2 \pm \sqrt{2}}{2}, 0\right)$ ii. (0,1) 

4. (a) $x = 1$ (b) (1,9)

(c) i. $\left(\frac{2 \pm 3\sqrt{2}}{2}, 0\right)$ ii. (0,7) 

5. (a) $k = \frac{9}{4}$ (b) $k < \frac{9}{4}$ (c) $k > \frac{9}{4}$ **6.** (a) $k = \frac{25}{8}$

(b) $k < \frac{25}{8}$ (c) $k > \frac{25}{8}$ **7.** (a) $k = \pm 1$ (b) $-1 < k < 1$ (c) $k < -1 \cup k > 1$

8. (a) $y = \frac{5}{12}(x-2)(x-6)$ (b) $y = -\frac{3}{8}(x+4)^2$ (c) $y = \frac{3}{4}(x-2)^2 + 1$ (d) $y = 3x^2 - 6x + 7$

9. (a) $y = -\frac{2}{5}x(x-6)$ (b) $y = \frac{3}{4}(x-3)^2$ (c) $y = \frac{7}{9}(x+2)^2 + 3$ (d) $y = -\frac{7}{3}x^2 - 2x + \frac{40}{3}$

EXERCISE 2.4.3

1. (a) $]-\infty,-2[\cup]1,\infty[$ (b) $[-3,2]$ (c) $]-\infty,0]\cup[4,\infty[$ (d) $]\frac{1}{3},3[$ (e) $]-\infty,-1.5]\cup[-1,\infty[$ (f) $]0.75,2.5[$

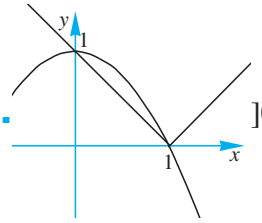
2. (a) $]-\infty,-2[\cup]-1,\infty[$ (b) $]-2,3[$ (c) $]-\infty,-0.5]\cup[3,\infty[$ (d) $[-2,2]$ (e) $]\frac{-1-\sqrt{21}}{2}, \frac{-1+\sqrt{21}}{2}[$

(f) $]-\infty,-2]\cup[3,\infty[$ (g) $[\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}]$ (h) $[-2.5,1]$ (i) $]-\infty,-3[\cup]0.5,\infty[$ (j) $]1,3[$ (k) $]-1,0.5[$ (l) Ø

(m) Ø (n) $[-1.5,5]$ (o) $]-\infty,-2[\cup]\frac{1}{3},\infty[$ **3.** (a) $-1 < k < 0$ (b) $-2\sqrt{2} < k < 2\sqrt{2}$ (c) $n \le -0.5$

4. (a) i. $]-\infty,-1[\cup]2,\infty[$ ii. $[-1,2]$ (b) i. $]-\infty,2[\cup]3,\infty[$ ii. $[2,3]$ (c) i. $]1,3[$ ii. $]-\infty,1]\cup[3,\infty[$

(d) i. $]-\frac{2}{3},1[$ ii. $]-\infty,-\frac{2}{3}]\cup[1,\infty[$ (e) i. $]-\infty,-2[\cup]2,\infty[$ ii. $[-2,2]$ (f) i. $]2-\sqrt{3}, 2+\sqrt{3}[$

ii. $]-\infty, 2-\sqrt{3}[\cup]2+\sqrt{3}, \infty[$ **5.** 

$]0,1[$ **6.** $[-2,0.5]$

7. (a) $\{x: x < -3\} \cup \{x: x > 2\}$ (b) $\{x: -1 < x < 4\}$

EXERCISE 2.4.4

1. (a) (–2,–3) (2,5) (b) (–2,–1) (1,2) (c) $\left(-\frac{1}{3}, -2\right), (2, 5)$ (d) $\left(-\frac{3}{2}, -\frac{15}{4}\right), (1, 0)$

(e) $\left(-\frac{9}{2}, -\frac{19}{4}\right), (1, -2)$ (f) $\left(\frac{3+\sqrt{73}}{4}, \frac{-3-\sqrt{73}}{8}\right), \left(\frac{3-\sqrt{73}}{4}, \frac{-3+\sqrt{73}}{8}\right)$

(g) $\left(\frac{1-\sqrt{13}}{2}, 1-\sqrt{13}\right), \left(\frac{1+\sqrt{13}}{2}, 1+\sqrt{13}\right)$ (h) no real solutions

(i) $\left(\frac{1-\sqrt{17}}{2}, \frac{5-3\sqrt{17}}{2}\right), \left(\frac{1+\sqrt{17}}{2}, \frac{5+3\sqrt{17}}{2}\right)$ (j) (–2,–3), (2,1) (k) no real solutions

2. (a) (1,4), (–7,84) (b) $\left(\frac{4}{3}, -\frac{56}{9}\right), \left(\frac{3}{4}, -\frac{7}{4}\right)$ (c) (0, 2), (3, 23) (d) $(-a, -a^2), \left(\frac{a}{2}, \frac{a^2}{2}\right)$ (e) Ø

(f) (2,8) (g) Ø (h) $\left(\frac{1}{2}, \frac{23}{4}\right)$ **3.** (a) $\pm 2\sqrt{6}$ (b) $m < -2\sqrt{6}, m > 2\sqrt{6}$ (c) $-2\sqrt{6} < m < 2\sqrt{6}$

4. $\sqrt{80}$ **5.** 1.75 **7.** $-\frac{23}{12}$ **8.** $c = \frac{a}{m}$ **10.** (a) i. (1,3), $\left(-\frac{14}{3}, \frac{196}{3}\right)$ ii. (–2, 12), $\left(\frac{7}{3}, \frac{49}{3}\right)$

(c) i. A(1,3), B(–2,2); $3y = x + 8$ ii. 4 sq. units

EXERCISE 3.1.1

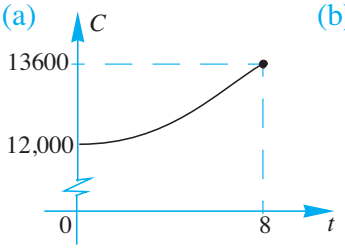
1. 8 **2.** 4, 0.25 **3.** 8, 18 **4.** 8 and 11 or –8 and –11 **5.** 6, –10 **6.** 2 m **7.** 51 kmh^{-1} **8.** 11, 13; –11, –13 **9.** 25 days **10.** 30 **11.** (a) 30 (b) \$50 each. **12.** 6 kmh^{-1} **13.** 16 **14.** 6 **15.** 3 hours **16.** 9 **17.** (a) 15 hrs (b) 10 hrs **18.** Chair-one: 20; Chair-two: 24 **19.** (a) 2 km (b) 2.5 km **20.** 7.5 hrs, 10.5hrs

EXERCISE 3.1.2

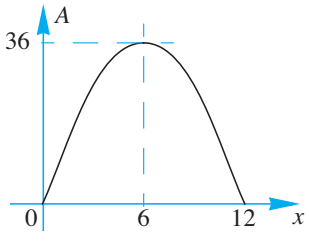
1. (a) i. $100 - 2x$ ii. $0 < x < 50$ [Nb: if $x = 0$ or 50, $A = 0$ and so there is no enclosure]

(b) i. $A = 2x(50 - x), 0 < x < 50$ ii. 10m by 80m or 40m by 20m iii. 1250 m^2 iv. 25m by 50 m

2. (a) ii. $0 < x < 12$ (b) i. 20 m^2 ii. 32 m^2 iii. 32 m^2 (c) (d) 6 m by 6 m

3. (a) 

(b) \$12,900

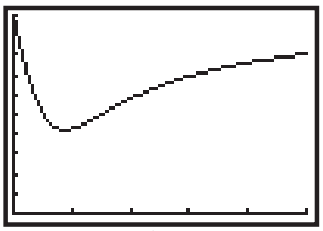


4. (a) $R(x) = xp = x(40 - 0.0004x), 0 \le x \le 100,000$ (b) i. \$960,000 ii. 18377 or 81622 [as answer must be integer values] iii. \$1,000,000

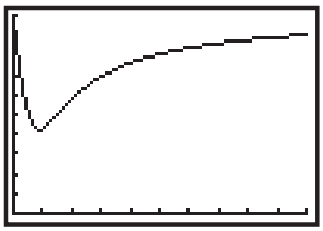
5. (a) $y = \frac{4}{3}(50 - x)$ (b) i. $A = \frac{8}{3}x(50 - x)$ ii. $0 < x < 50$ (c) i. $\frac{5000}{3}$ m^2

ii. $x = 25, y = \frac{200}{3}$; dimensions 50 m by $\frac{100}{3}$ m. **6.** 35.83 kmh^{-1} **7.** (a) 100%

(b) $t = 0.229$ (first time) then again at $t = 13.104$ (c) (d) i. 42.26% ii. 1.73 weeks



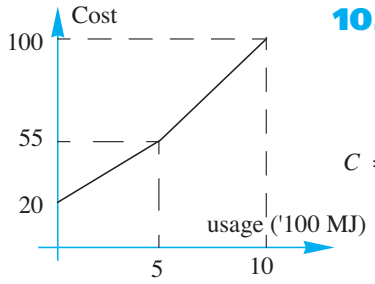
[0, 10] by [0, 1]



[0, 20] by [0, 1]

(e) As time increases, oxygen level will be 100%

8. (a) (b) \$95.26

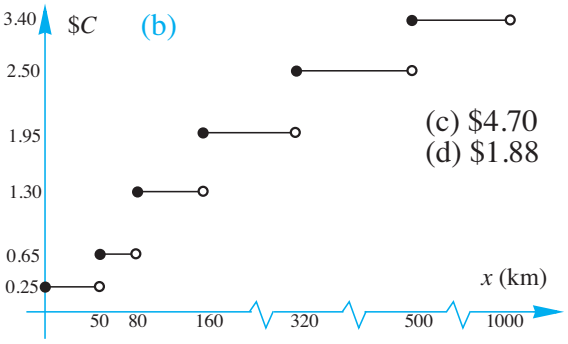


9. B(3254, 1953), C(6146,3687) units in metres

10. (a)

$$C = \begin{cases} 0.25, & 0 \le x < 50 \\ 0.65, & 50 \le x < 80 \\ 1.30, & 80 \le x < 160 \\ 1.95, & 160 \le x < 320 \\ 2.50, & 320 \le x < 500 \\ 3.40, & 500 \le x < 1000 \end{cases}$$

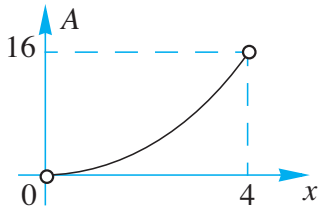
(b)



(c) \$4.70
(d) \$1.88

11. (a) $0 < x < 4$ (b) $A(x) = 3x + 0.25x^2, 0 < x < 4$ (c) i. ii. 3.25 iii. $-6 + \sqrt{48}$

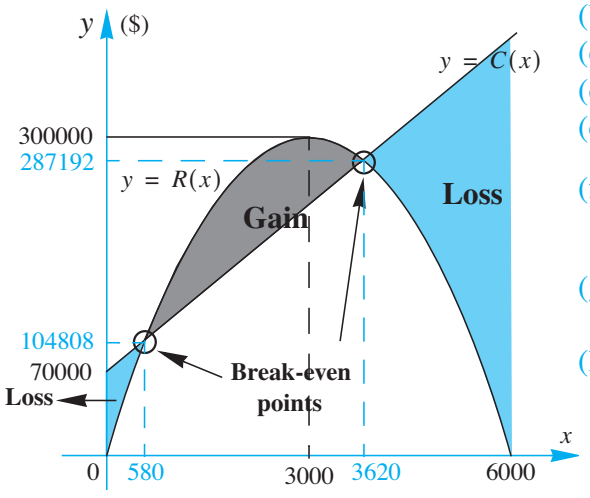
12. (a) i. 200 m ii. 320 m
(b) i. 0.34 sec and 11.66 sec ii. 11.31 sec (c) 12 sec
(d) 360 m



13. (a) 0.53 sec (on the way up) and 9.47 sec (on the way down)
(b) 10 sec (c) 500 m (d) 12.07 sec (e) 750 m

14. (a) \$72500 (b) No. (Loss of \$20000) (c) 2500

15. (a)

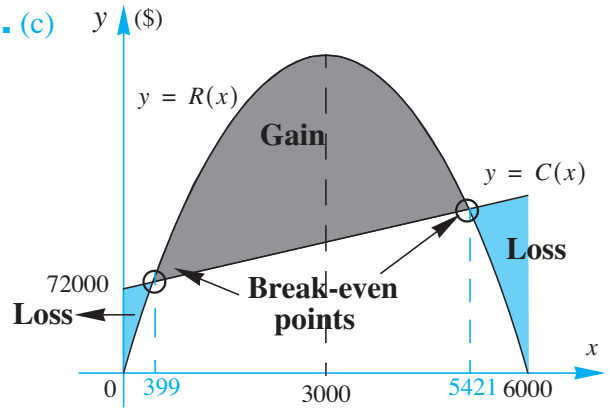


(b) i. \$70,000 ii. \$300,000
(c) Fixed cost (e.g., salary, electricity, ...)
(d) See graph in (a)
(e) \$76667 (to nearest dollar)
(f) i. $P(x) = 140x - \frac{1}{30}x^2 - 70000, 0 \le x \le 6000$
ii. \$77,000 iii. 2100
(g) i. $0 \le x \le 580$ or $3620 \le x \le 6000$
ii. $581 \le x \le 3619$
(h) See graph in (a)

16. (b) i. $P(x) = -\frac{1}{30}x^2 + 194x - 72000$ ii. $0 \le x \le 6000$

(d) The company will break-even at 399 radios and 5421 radios. Provided the company sells between 399 and 5421 radios they will make a profit. (e) 2910

10. (c)

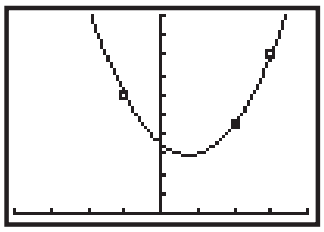


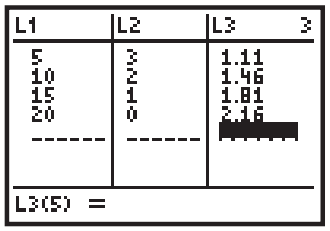
EXERCISE 3.1.3

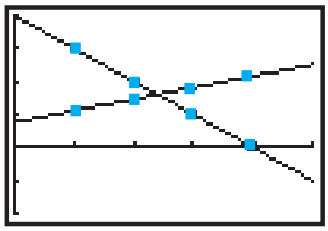
1. i. (b) $y = 0.4x + 7.2$ ii. (b) $y = 6 - 2x$ iii. (b) $y = 0.5x + 3.2$ **2.** Second difference = 0.64

3. (b) $y = x^2 + 4x + 2$ **4.** (a) & (c)

(b) $y = 2x^2 - 3x + 7$ **5.** $y = 2x^2 - x + 3$ **6.**







(c) i. $p = -0.2q + 4$ ii. $p = 0.07q + 0.76$

(d) Optimium scenario: demand = supply. This occurs when $p = 1.60$, $q = 12$.

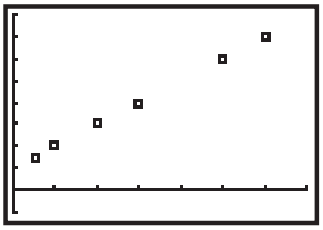
7. (a) $y = -0.6333x^2 + 8.833x - 19.2$

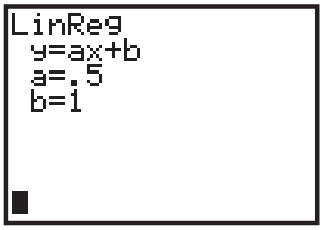
(b) –40°C at 11 pm.

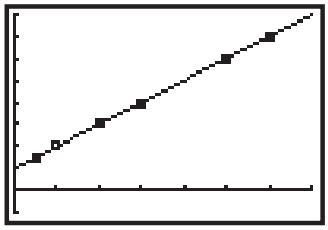
The model is not valid outside data range. Therefore extrapolation will not necessarily work.

8. Equation of path: $y = -\frac{31}{2400}x^2 + \frac{49}{48}x + 1$. Greatest height: 21.17 m.

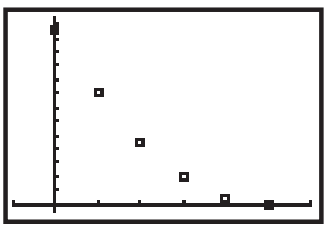
9. (a) (b) Linear (c) i. ii. $M = 0, x = 1$. i.e., 1 m

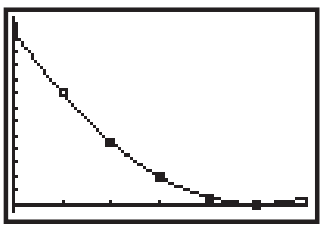


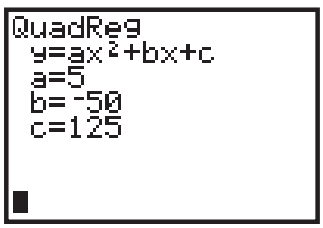




10. (a) (b) (c) i. parabola (d)



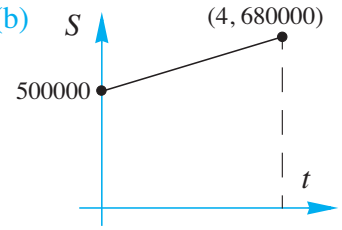




11. (a) $P(x) = -2x^2 + x + 3$ (b) $P(x) = (1-k)x^2 + x + k,\ x \in \mathbb{R},\ k \neq 1$

12. (a) i. $a = 45000, b = 500000$ ii. $k_1 = 0, k_2 = 4$

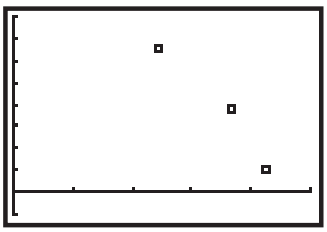
(b)

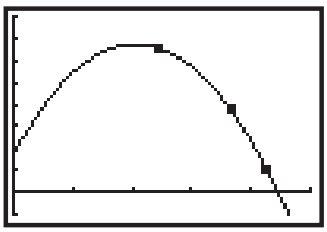


13. (a) (b) i. Parabolic

ii. $h(x) = -0.04694x^2 + 0.96518x + 1.7896$

(c) i. 6.75 m ii. 22.27 m

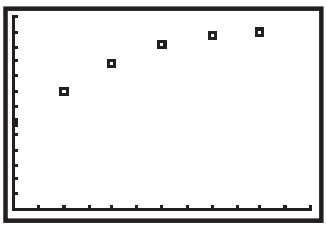




14. (a) $y = -\frac{2}{9}x^2 + \frac{11}{9}x + \frac{9}{2},\ 0 \le x \le 5.5$ (b) [BE] : $y = -\frac{11}{9}x + \frac{121}{18},\ 1 \le x \le 5.5$,

[BO] : $y = 5.5x,\ 0 \le x \le 1$ (c) $49°36'$

15. (a)



(b) second difference is constant = –50

(c) $y = -0.25x^2 + 25x + 580$ (d) \$22500 per car (e) i. \$824,750 ii. \$19750

16. (a) i. & ii. have a constant gradient. iii. results imply quadratic form (b) ii. $p = 10 - 0.001x$, $C(x) = 2x + 7000$,

$R(x) = -0.001x^2 + 10x$.

(c) $P(x) = -0.001x^2 + 8x - 7000$, max profit = $P(4000) = 9000$

EXERCISE 4.1.1

1. (a) $b^2 + 2bc + c^2$ (b) $a^3 + 3a^2g + 3ag^2 + g^3$ (c) $1 + 3y + 3y^2 + y^3$

(d) $16 + 32x + 24x^2 + 8x^3 + x^4$ (e) $8 + 24x + 24x^2 + 8x^3$ (f) $8x^3 - 48x^2 + 96x - 64$

(g) $16 + \frac{32}{7}x + \frac{24}{49}x^2 + \frac{8}{343}x^3 + \frac{1}{2401}x^4$ (h) $8x^3 - 60x^2 + 150x - 125$

(i) $27x^3 - 108x^2 + 144x - 64$ (j) $27x^3 - 243x^2 + 729x - 729$ (k) $8x^3 + 72x^2 + 216x + 216$

(l) $b^3 + 9b^2d + 27bd^2 + 27d^3$ (m) $81x^4 + 216x^3y + 216x^2y^2 + 96xy^3 + 16y^4$

(n) $x^5 + 15x^4y + 90x^3y^2 + 270x^2y^3 + 405xy^4 + 243y^5$ (o) $\frac{125}{p^3} + \frac{150}{p} + 60p + 8p^3$

(p) $\frac{16}{x^4} - \frac{32}{x} + 24x^2 - 8x^5 + x^8$ (q) $q^5 + \frac{10q^4}{p^3} + \frac{40q^3}{p^6} + \frac{80q^2}{p^9} + \frac{80q}{p^{12}} + \frac{32}{p^{15}}$ (r) $x^3 + 3x + \frac{3}{x} + \frac{1}{x^3}$

EXERCISE 4.1.2

1. (a) $160x^3$ (b) $21x^5y^2$ (c) $-448x^3$ (d) $-810x^4$ (e) $216p^4$ (f) $-20412p^2q^5$ (g) $-22680p$

2. (a) –1400000 (b) 6000 (c) 540 (d) –240 (e) 81648 (f) 40 **3.** 1.0406 0.0004%

4. i. $64x^6 + 960x^5 + 6000x^4 + 20000x^3 + 37500x^2 + 37500x + 15625$ ii. 19750 iii. 20.6

iv. 0.1% **5.** 19 **6.** $-\frac{63}{8}$ **7.** $\frac{231}{16}$ **8.** $-\frac{130}{27}$ **9.** –20 **10.** $a = \pm 3$ **11.** $n = 5$ **12.** $n = 9$

13. (a) 0 (b) –59 **14.** $a = 3, n = 8$ **15.** $a = \pm 2, b = \pm 1$

EXERCISE 5.1

1. (a) dom = {2, 3, –2}, ran = {4, –9, 9} (b) dom = {1, 2, 3, 5, 7, 9}, ran = {2, 3, 4, 6, 8, 10}
(c) dom = {0, 1}, ran = {1, 2} **2.** (a) $]1, \infty[$ (b) $[0, \infty[$ (c) $]9, \infty[$ (d) $]-\infty, 1]$ (e) $[-3, 3]$
(f) $]-\infty, \infty[$ (g) $]-1, 0]$ (h) $[0, 4]$ (i) $[0, \infty[$ (j) $[1, 5]$ (k) $]0, 4[$ (l) $]-\infty, -1] \cup [1, \infty[$
3. (a) $r = [-1, \infty[$, $d = [0, 2[$ (b) $r = \{y : y \geq 0\} \setminus \{4\}$, $\mathbb{R}$ (c) $r = [0, \infty[\setminus \{3\}$, $d = [-4, \infty[\setminus \{0\}$
(d) $r = [-2, 0[$, $d = [-1, 2[$ (e) $r =]-\infty, \infty[$ $d =]-\infty, -3] \cup [3, \infty[$ (f) $r = [-4,4]$, $d = [0,8]$
4. (a) one to many (b) many to one (c) many to one (d) one to one (e) many to many
(f) one to one **5.** (a) $\mathbb{R} \setminus \{-2\}$ (b) $]-\infty, 9[$ (c) $[-4,4]$ (d) $]-\infty, -2] \cup [2, \infty[$ (e) $\mathbb{R} \setminus \{0\}$ (f) $\mathbb{R}$
(g) $\mathbb{R} \setminus \{-1\}$ (h) $[-a, \infty[$ (i) $[0, \infty[\setminus \{a^2\}$ (j) $]-\infty, -a] \cup [a, \infty[$ (k) $\mathbb{R}$ (l) $\mathbb{R} \setminus \{-a^{-1}\}$
6. (a) $]-\infty, -a[$ (b) $]0, ab]$ (c) $]-\infty, \frac{1}{4}a^3]$ (d) $[\frac{1}{4}a^3, \infty[$ (e) $\mathbb{R} \setminus \{a\}$ (f) $]-\infty, a[$ (g) $[-a, \infty[$ (h) $]-\infty, 0[$

EXERCISE 5.2

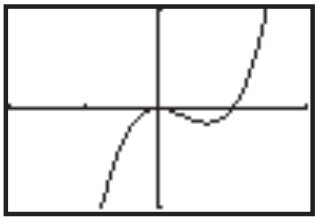
Graphs with graphics calculator output have standard viewing window unless otherwise stated.

1. (a) 3, 5 (b) i. $2(x+a) + 3$ ii. $2a$ (c) 3 **2.** (a) 0, $\frac{10}{11}$ (b) $-\frac{5}{4}$ (c) $\left[0, \frac{10}{11}\right]$

3. (a) $-\frac{1}{2}x^2 - x + \frac{3}{2}$, $-\frac{1}{2}x^2 + x + \frac{3}{2}$ (b) $\pm\sqrt{2}$ (c) no solution

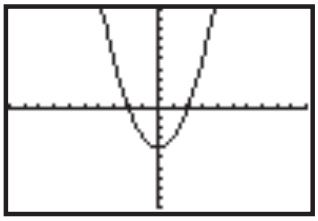
4. (a) $x = 0, 1$

(b)

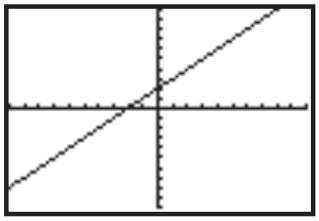


Window $[-2,2]$, $[-1,1]$
Range: $[-12, 4]$

5. (a) i.



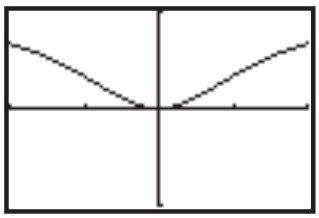
ii.



(b) i. $\{2\sqrt{2}, -2\sqrt{2}\}$ ii. $\{3, -2\}$

6. (b), (c), (d), (e) **8.** (a), (d), (e), (f)

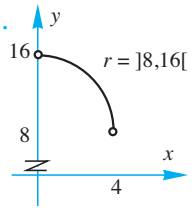
9. (a)



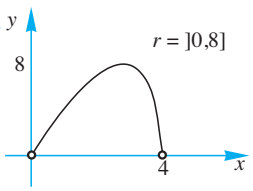
Window $[-2,2]$, $[-1,1]$
(b) $[0, 1[$

10. (a) $\{y : y > 1\} \cup \{y : y \leq -1.25\}$ (b) 10 **11.** (b) 1
12. (a) only - it is the only one with identical rules & domains.
13. (a) $[-3, \infty[$ (b) $[-3,0]$ (c) $[3, \infty[$ (d) $[1.5,3[\cup]3,\infty[$
14. (a) i. $p(x) = 8 + 2\sqrt{16 - x^2}$, $0 < x < 4$
ii. $A(x) = x\sqrt{16 - x^2}$, $0 < x < 4$

(b) i.

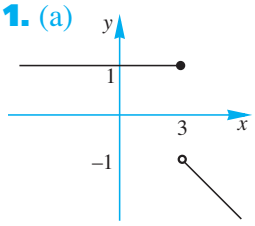


ii.

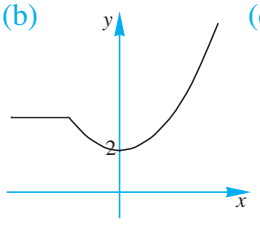


EXERCISE 5.3.1

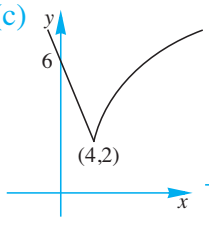
1. (a)



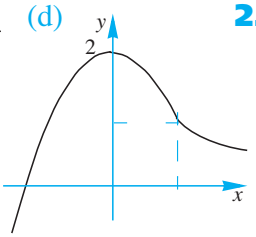
(b)



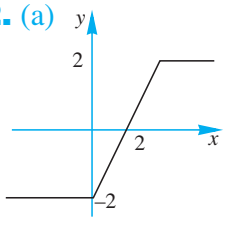
(c)



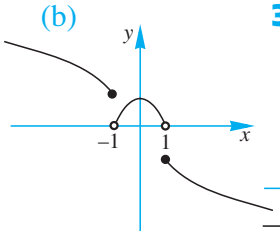
(d)



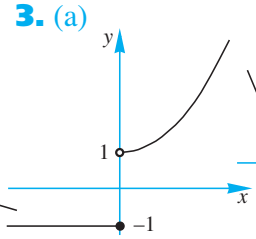
2. (a)



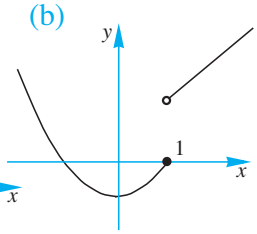
(b)



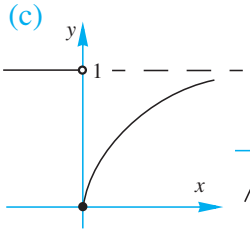
3. (a)



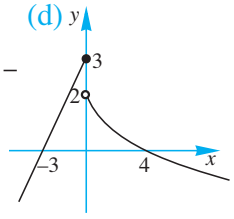
(b)



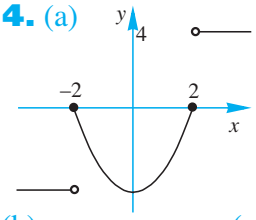
(c)



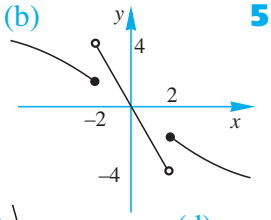
(d)



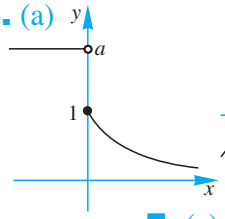
4. (a)



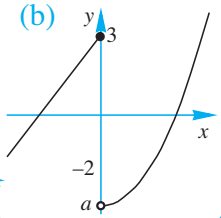
(b)



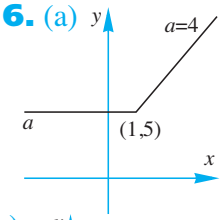
5. (a)



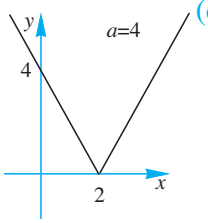
(b)



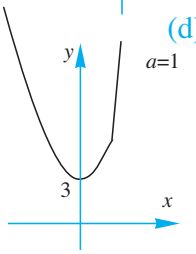
6. (a)



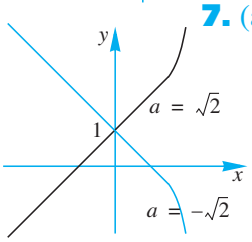
(b)



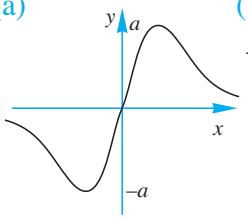
(c)



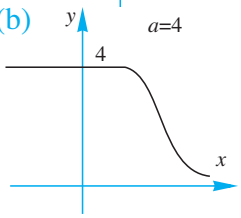
(d)



7. (a)

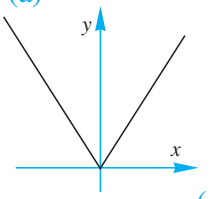


(b)

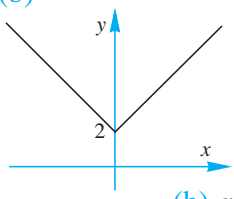


EXERCISE 5.3.2

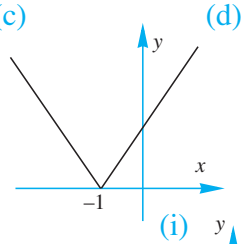
1. (a)



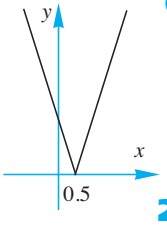
(b)



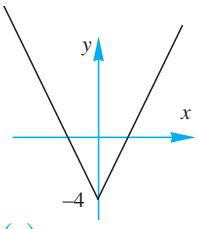
(c)



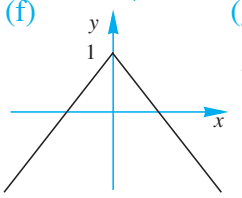
(d)



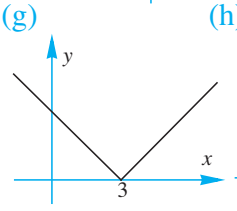
(e)



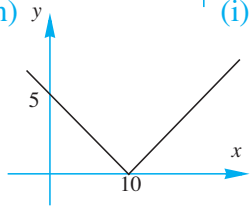
(f)



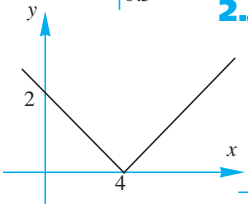
(g)



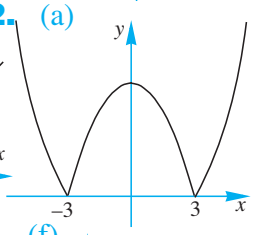
(h)



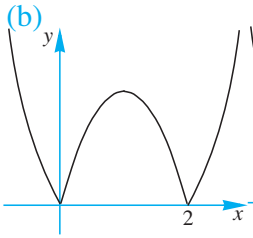
(i)



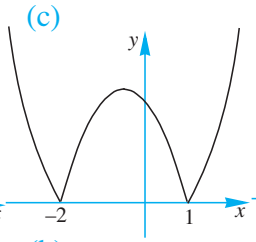
2. (a)



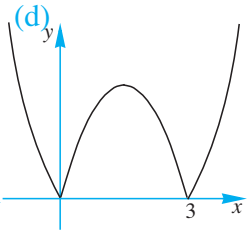
(b)



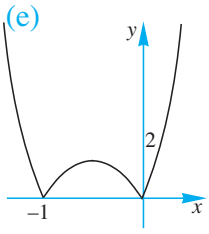
(c)



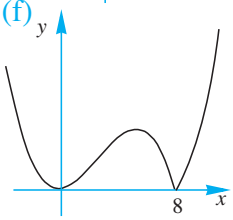
(d)



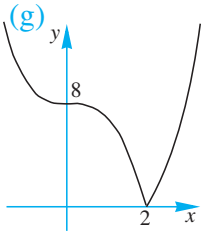
(e)



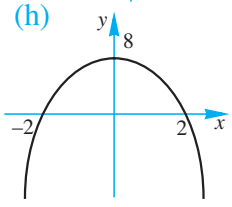
(f)



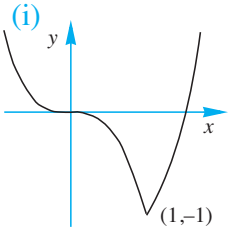
(g)



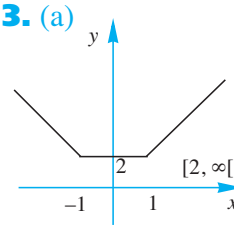
(h)



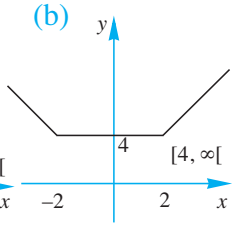
(i)



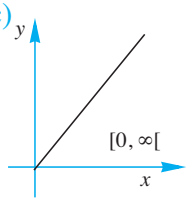
3. (a)



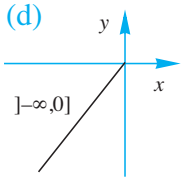
(b)



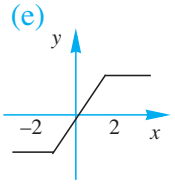
(c)



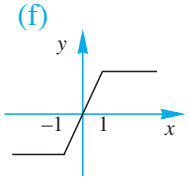
(d)

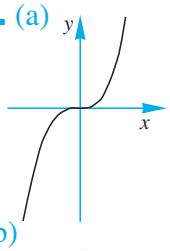


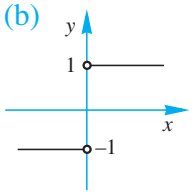
(e)

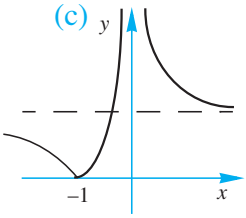


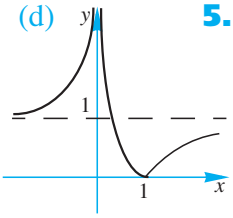
(f)

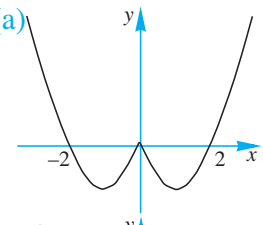


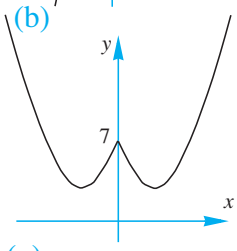
4. (a) 

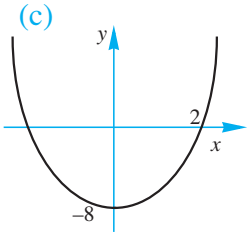
(b) 

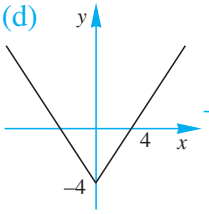
(c) 

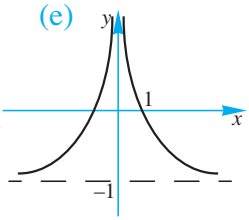
(d) 

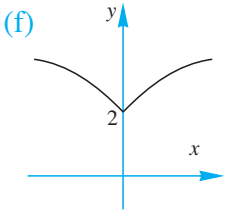
5. (a) 

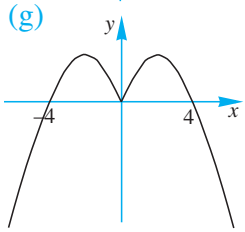
(b) 

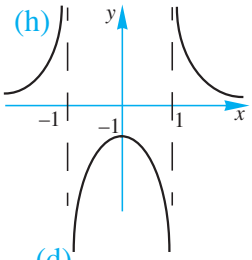
(c) 

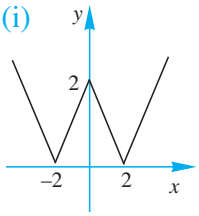
(d) 

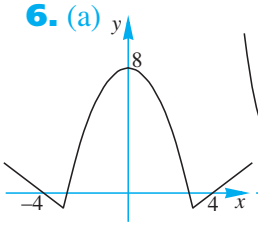
(e) 

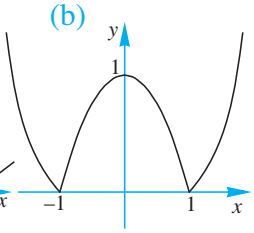
(f) 

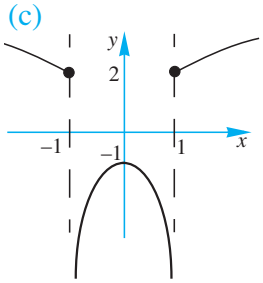
(g) 

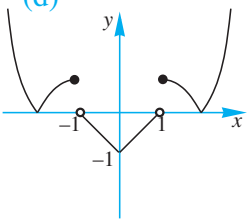
(h) 

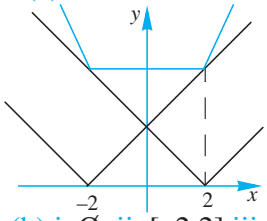
(i) 

6. (a) 

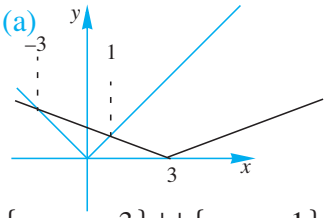
(b) 

(c) 

(d) 

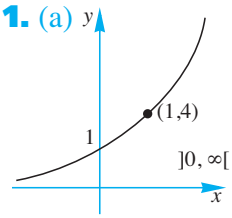
7. (a) 

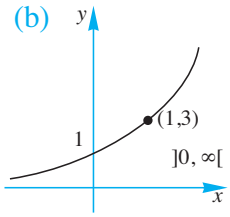
(b) i. Ø ii. [–2,2] iii. {±4}

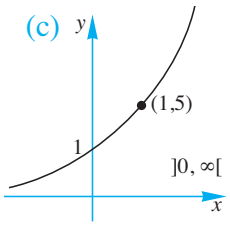
8. (a) 

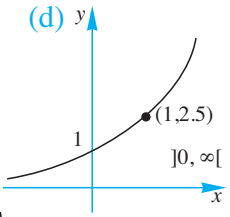
$\{x: x \leq -3\} \cup \{x: x \geq 1\}$

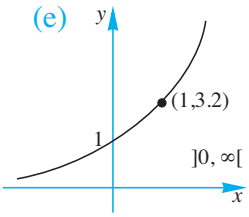
EXERCISE 5.3.3

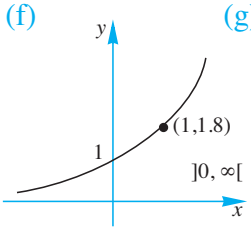
1. (a) 

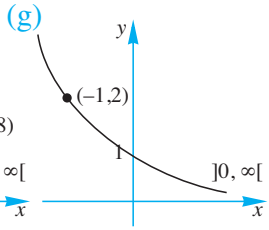
(b) 

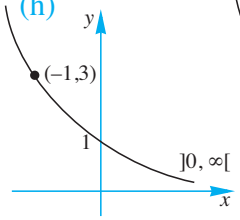
(c) 

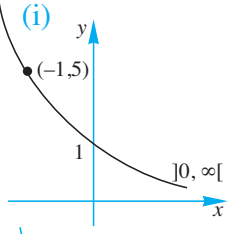
(d) 

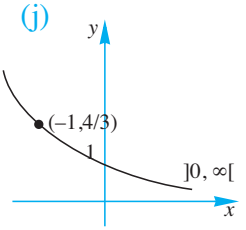
(e) 

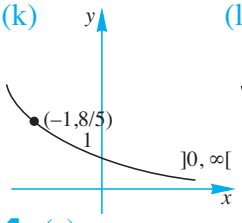
(f) 

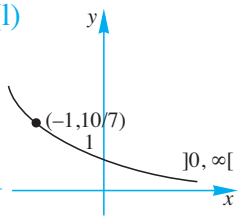
(g) 

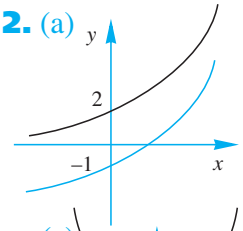
(h) 

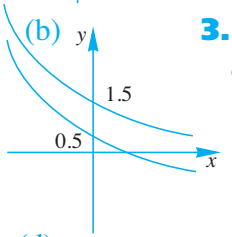
(i) 

(j) 

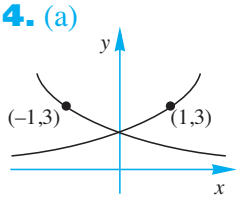
(k) 

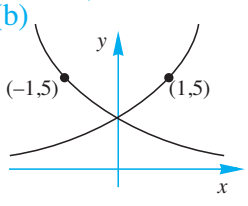
(l) 

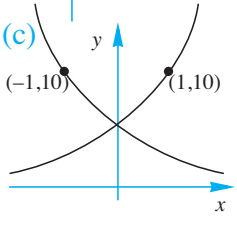
2. (a) 

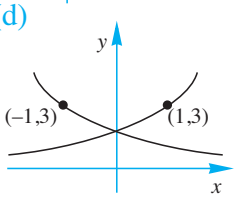
(b) 

3. '*b*' has a dilation effect on $f(x) = a^x$ (along the *y* axis).

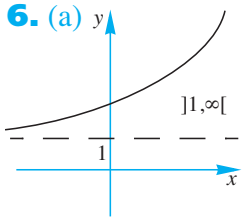
4. (a) 

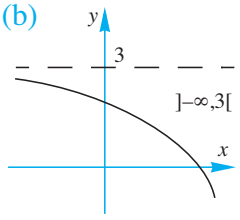
(b) 

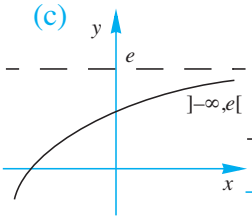
(c) 

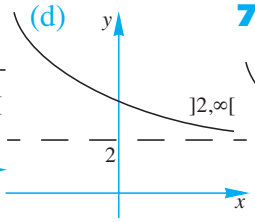
(d) 

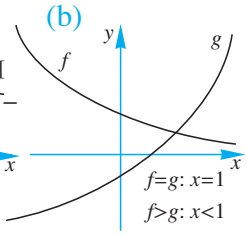
5. (a) [1, 16] (b) [3, 27] (c) [0.25, 16] (d) [0.5, 4] (e) [0.125, 0.25] (f) [0.1, 10]

6. (a) 

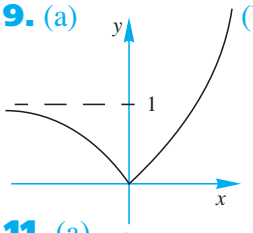
(b) 

(c) 

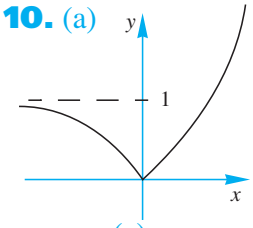
(d) 

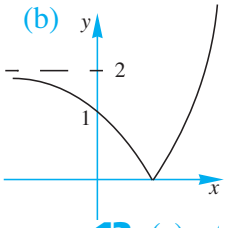
7. (a) –1. 5
(b) 

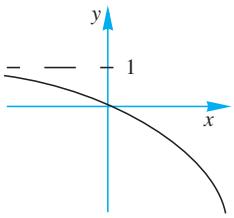
8. (a) $]2,2+e^{-1}[$ (b) $[-1,1[$ (c) $[1-e,1+e^{-1}]$

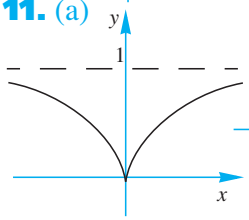
9. (a) 

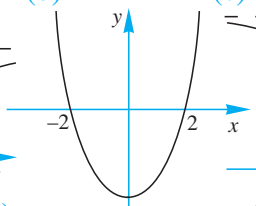
(b) 2

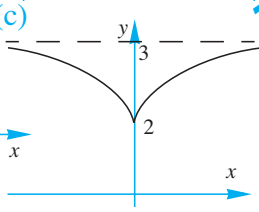
10. (a) 

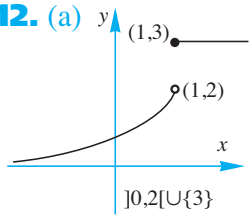
(b) 

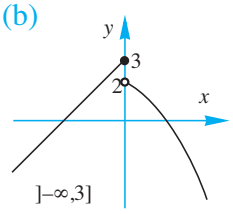
(c) 

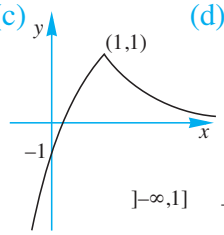
11. (a) 

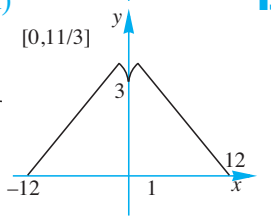
(b) 

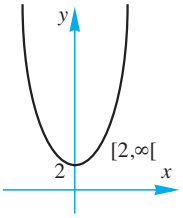
(c) 

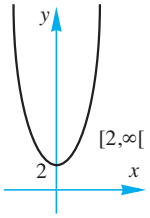
12. (a) 

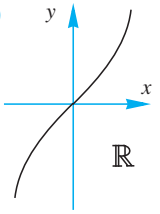
(b) 

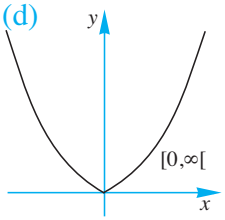
(c) 

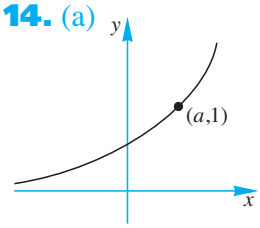
(d) 

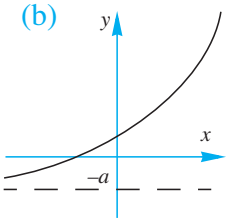
13. (a) 

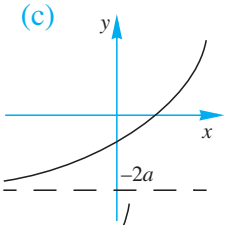
(b) 

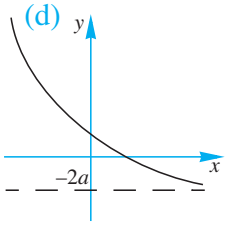
(c) 

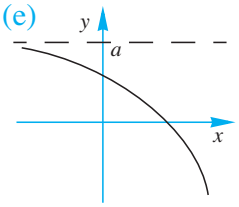
(d) 

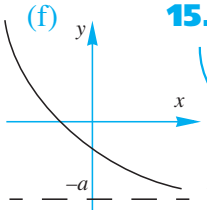
14. (a) 

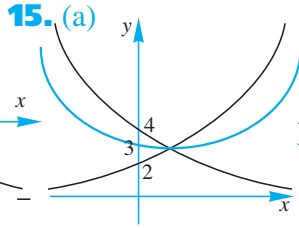
(b) 

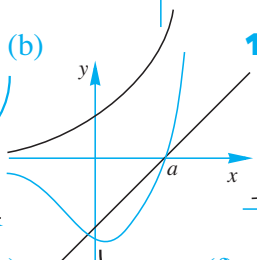
(c) 

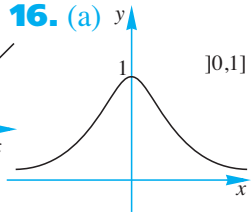
(d) 

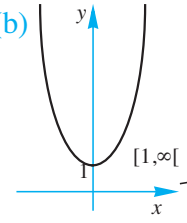
(e) 

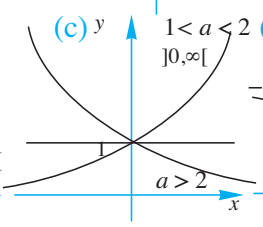
(f) 

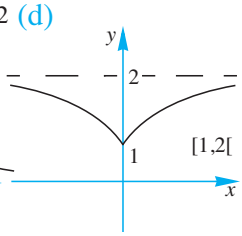
15. (a) 

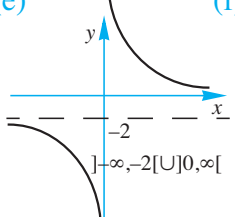
(b) 

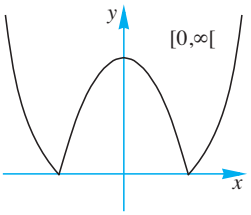
16. (a) 

(b) 

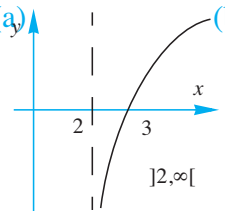
(c) 

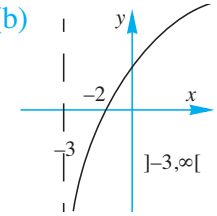
(d) 

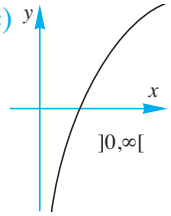
(e) 

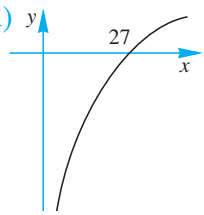
(f) 

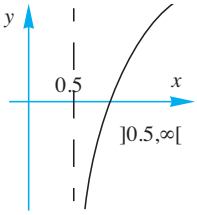
EXERCISE 5.3.4

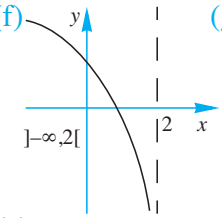
1. (a) 

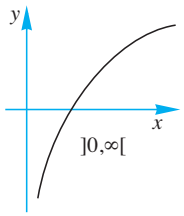
(b) 

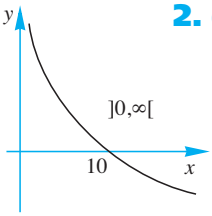
(c) 

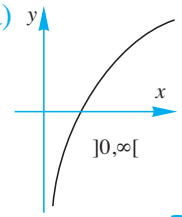
(d) 

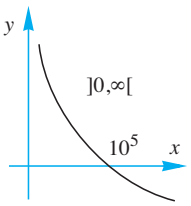
(e) 

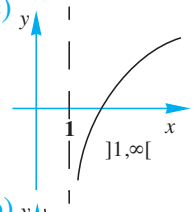
(f) 

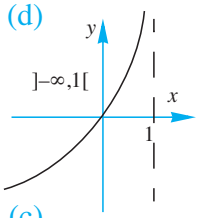
(g) 

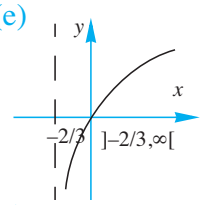
(h) 

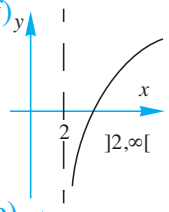
2. (a) 

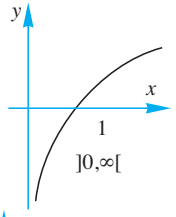
(b) 

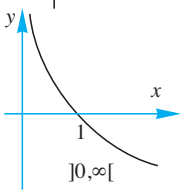
(c) 

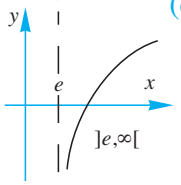
(d) 

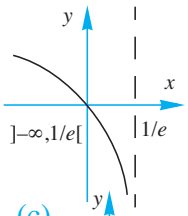
(e) 

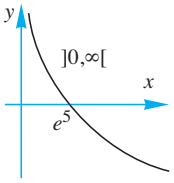
(f) 

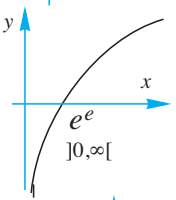
3. (a) 

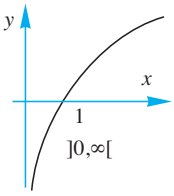
(b) 

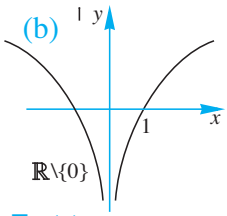
(c) 

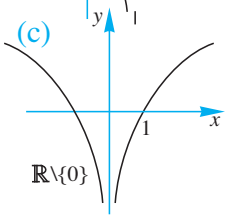
(d) 

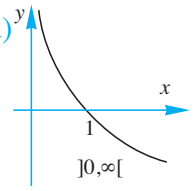
(e) 

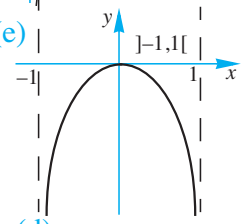
(f) 

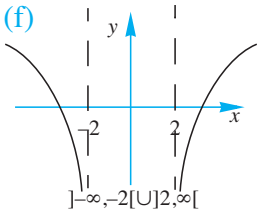
4. (a) 

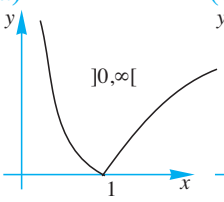
(b) 

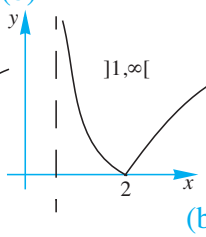
(c) 

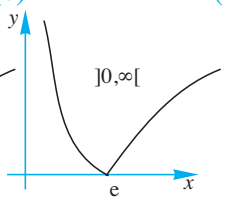
(d) 

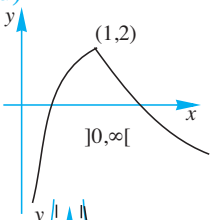
(e) 

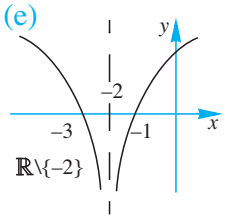
(f) 

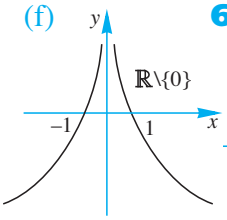
5. (a) 

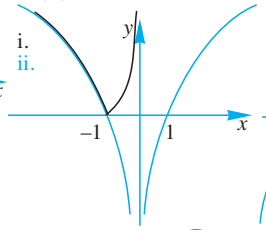
(b) 

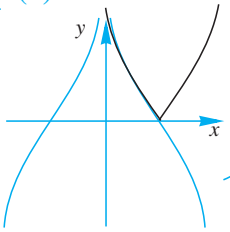
(c) 

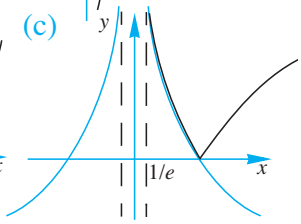
(d) 

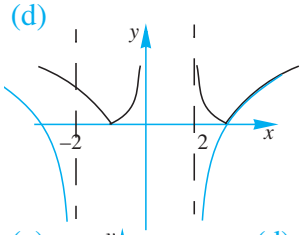
(e) 

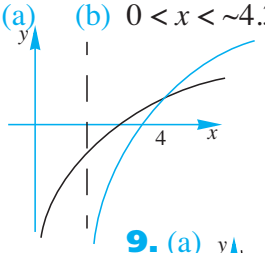
(f) 

6. (a) 

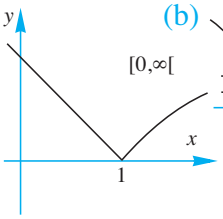
(b) 

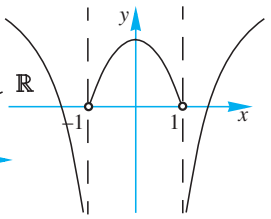
(c) 

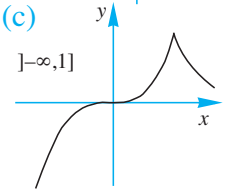
(d) 

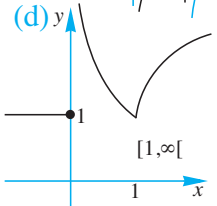
7. (a) 

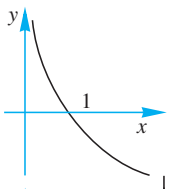
(b) $0 < x < \sim 4.3$

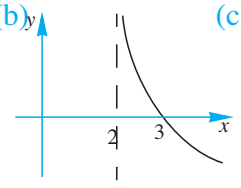
8. (a) 

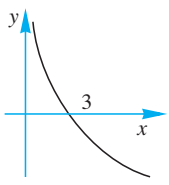
(b) 

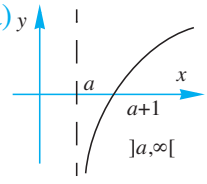
(c) 

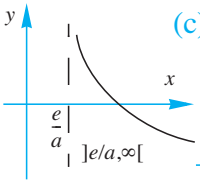
(d) 

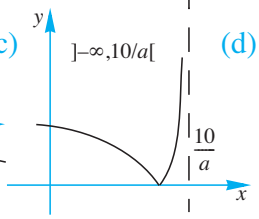
9. (a) 

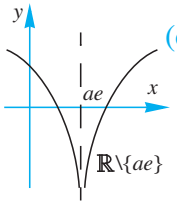
(b) 

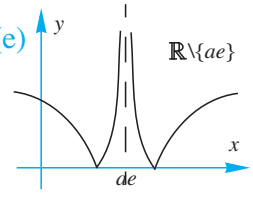
(c) 

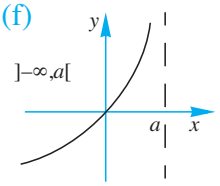
10. (a) 

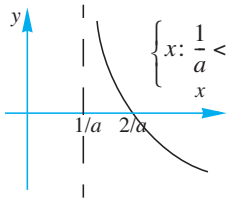
(b) 

(c) 

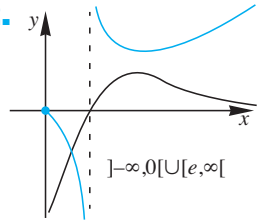
(d) 

(e) 

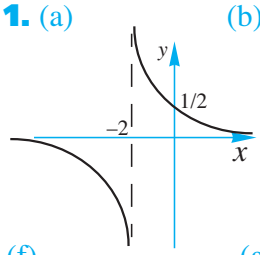
(f) 

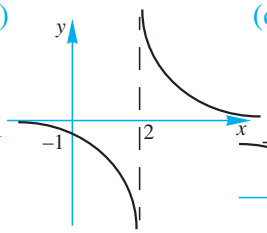
11. (a) 

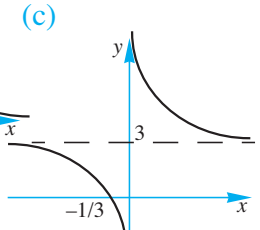
$\left\{x: \frac{1}{a} < x < 1 + \frac{1}{a}\right\}$

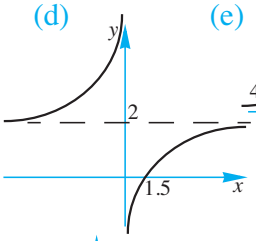
12. 

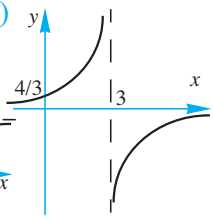
EXERCISE 5.3.5

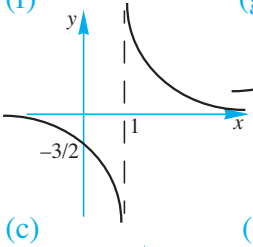
1. (a) 

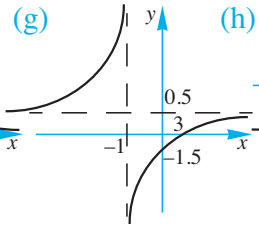
(b) 

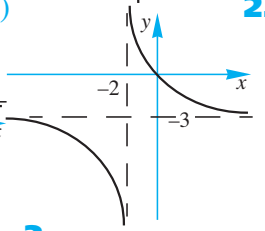
(c) 

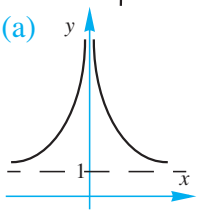
(d) 

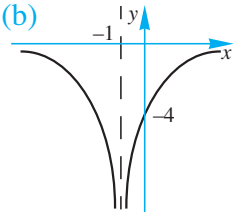
(e) 

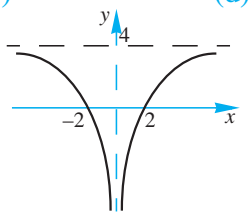
(f) 

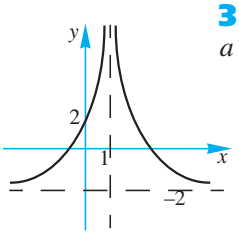
(g) 

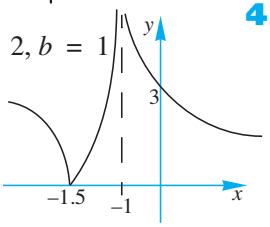
(h) 

2. (a) 

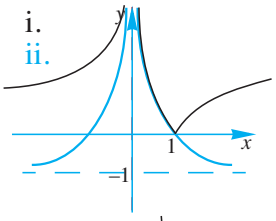
(b) 

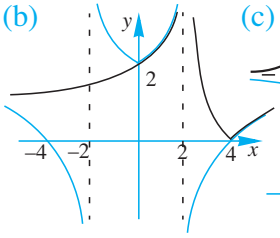
(c) 

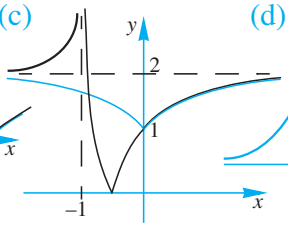
(d) 

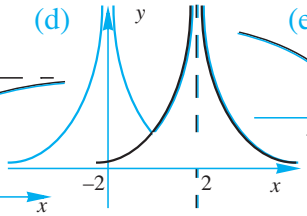
3. $a = 2, b = 1$ 

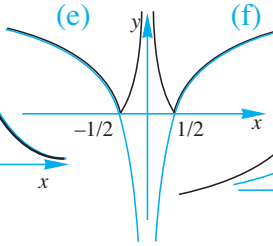
4. (a) i.

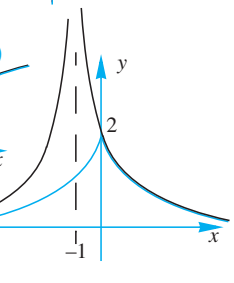
ii. 

(b) 

(c) 

(d) 

(e) 

(f) 

EXERCISE 5.4.1

1. (a) i. $f+g: [0,\infty[\longmapsto \mathbb{R}$ where $(f+g)(x) = x^2 + \sqrt{x}$ $[0,\infty[$

ii. $f+g:]0,\infty[\longmapsto \mathbb{R}$ where $(f+g)(x) = \frac{1}{x} + \ln(x)$ $]-\infty,\infty[$

iii. $f+g: [-3,-2] \cup [2,3] \longmapsto \mathbb{R}$ where $(f+g)(x) = \sqrt{9-x^2} + \sqrt{x^2-4}$ $, [\sqrt{5}, \sqrt{10}]$

(b) i. $fg: [0,\infty[\longmapsto \mathbb{R}$ where $(fg)(x) = x^2\sqrt{x} = x^{5/2}$

ii. $fg:]0,\infty[\longmapsto \mathbb{R}$ where $(fg)(x) = \frac{\ln(x)}{x}$

iii. $fg: [-3,-2] \cup [2,3] \longmapsto \mathbb{R}$ where $(fg)(x) = \sqrt{(9-x^2)(x^2-4)}$

2. (a) i. $f-g:]-\infty,\infty[\longmapsto \mathbb{R}$ where $(f-g)(x) = 2e^x - 1$ $,]-1,\infty[$

ii. $f-g:]-1,\infty[\longmapsto \mathbb{R}$ where $(f-g)(x) = (x+1) - \sqrt{x+1}$ $,]-0.25,\infty[$

iii. $f-g:]-\infty,\infty[\longmapsto \mathbb{R}$ where $(f-g)(x) = |x-2| - |x+2|$ $, [-4,4]$

(b) i. $f/g: \mathbb{R} \setminus \{0\}, \longmapsto \mathbb{R}$ where $(f/g)(x) = \frac{e^x}{1-e^x}$

ii. f/g: $]-1, \infty[\longmapsto \mathbb{R}$ where $(f/g)(x) = \sqrt{x+1}$

iii. f/g: $\mathbb{R}\backslash\{-2\} \longmapsto \mathbb{R}$ where $(f/g)(x) = \left|\frac{x-2}{x+2}\right|$

3. i. (a) $fog(x) = x^3 + 1$, $gof(x) = (x+1)^3$ (b) $]-\infty, \infty[$, $]-\infty, \infty[$

ii. (a) $fog(x) = x + 1, x \geq 0$, $gof(x) = \sqrt{x^2+1}$ (b) $[1, \infty[$, $[1, \infty[$

iii. (a) $fog(x) = x^2$, $gof(x) = (x+2)^2 - 2$ (b) $[0, \infty[$, $[-2, \infty[$

iv. (a) $fog(x) = x, x \neq 0$, $gof(x) = x, x \neq 0$ (b) $\mathbb{R}\backslash\{0\}$, $\mathbb{R}\backslash\{0\}$

v. (a) $fog(x) = x, x \geq 0$, $gof(x) = |x|$ (b) $[0, \infty[$, $[0, \infty[$

vi. (a) $fog(x) = \frac{1}{x^2} - 1, x \neq 0$, $gof(x)$ does not exist (b) $]-1, \infty[$

vii (a) $fog(x) = x^2, x \neq 0$, $gof(x) = x^2, x \neq 0$ (b) $]0, \infty[$, $]0, \infty[$

viii. (a) $fog(x) = |x| - 4$, $gof(x) = |x-4|$ (b) $[-4, \infty[$, $[0, \infty[$

ix. (a) $fog(x) = |x+2|^3 - 2$, $gof(x) = |x^3|$ (b) $[-2, \infty[$, $[0, \infty[$

x. (a) $fog(x)$ does not exist, $gof(x) = (4-x), x \leq 4$ (b) $[0, \infty[$

xi. (a) $fog(x) = \frac{x^2}{x^2+1}$, $gof(x) = \left(\frac{x}{x+1}\right)^2, x \neq -1$ (b) $[0, 1[$, $[0, \infty[$

xii. (a) $fog(x) = x^2 + |x| + 1$, $gof(x) = |x^2 + x + 1|$ (b) $[1, \infty[$, $[0.75, \infty[$

xiii. (a) $fog(x) = 2^{x^2}$, $gof(x) = 2^{2x}$ (b) $[1, \infty[$, $]0, \infty[$

xiv. (a) $fog(x)$ does not exist, $gof(x) = \frac{1}{x+1} - 1, x \neq -1$ (b) $\mathbb{R}\backslash\{-1\}$

xv. (a) $fog(x)$ does not exist, $gof(x) = \frac{4}{x-1} + 1$ (b) $]1, \infty[$

xvi. (a) $fog(x) = 4^{\sqrt{x}}, x \geq 0$, $gof(x) = 4^{0.5x}$ (b) $[1, \infty[$, $]0, \infty[$

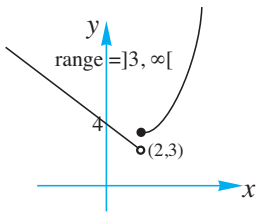
4. (a) $fog(x) = 2x + 3, x \in \mathbb{R}$ (b) $gof(x) = 2x + 2, x \in \mathbb{R}$ (c) $fof(x) = 4x + 3, x \in \mathbb{R}$

5. $g(x) = x^2 + 1, x \in \mathbb{R}$ **6.** (a) $fog(x) = \frac{1}{x} + x + 1, x \in \mathbb{R}\backslash\{0\}$,$]-\infty,-1] \cup [3,\infty[$

(b) $gof(x)$ does not exist. (c) $gog(x) = x + \frac{1}{x} + \frac{x}{x^2+1}, x \neq 0$, $]-\infty,-2.5] \cup [2.5,\infty[$

7. (a) 9 (b) 3 **9.** (a) $x = \pm 1$ (b) $x = 1, -3$ **10.** (a) $\frac{1}{x}$ (b) $\frac{-x}{2x+1}$

11. $hof(x) = \begin{cases} (x-1)^2 + 4, & x \geq 2 \\ 5 - x, & x < 2 \end{cases}$



12. (a) $r_f \subseteq d_g$ and $r_{gof} \subseteq d_h$ (b) $g(x) = 4(x+1)^2, x \in \mathbb{R}$

13. (a) $fog(x) = x, x \in]0,\infty[$ range $=]0,\infty[$

(b) $gof(x) = \frac{1}{2}(\ln(e^{2x-1}) + 1), x \in \mathbb{R}$ $(= x)$ range $=]-\infty,\infty[$

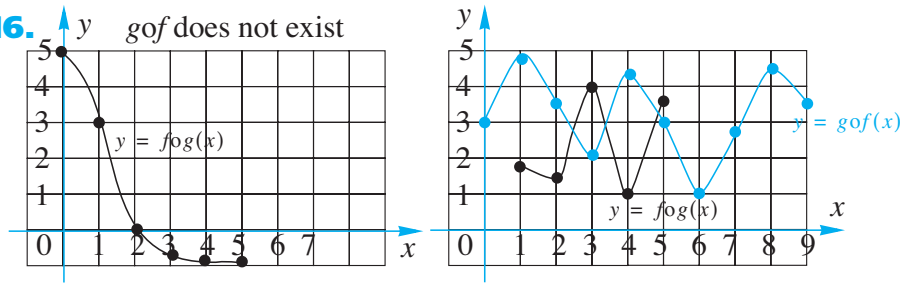
(c) $fof(x) = e^{2(e^{2x-1})-1}, x \in \mathbb{R}$ range $=]e^{-1},\infty[$

14. (a) hok does not exist.

(b) $koh(x) = 4\log(4x-1) - 1, x > \frac{1}{4}$, $\mathbb{R}$

15. (a) S $= \mathbb{R}\backslash]-3,3[$; T $= \mathbb{R}$ (b) T $= \{x : |x| \geq 6, x = 0\}$; S $=]-\infty,-3] \cup [3,\infty[$

16. *gof* does not exist

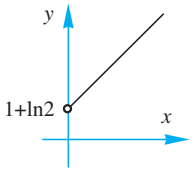


17. (a) Dom $f =]0,\infty[$, ran $f =]e,\infty[$, Dom $g =]0,\infty[$, ran $g = \mathbb{R}$

(b) *fog* does not exist: $r_g = \mathbb{R} \not\subset d_f =]0,\infty[$

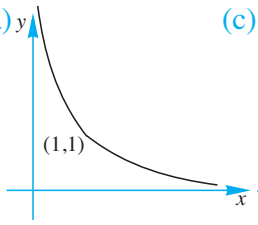
gof exists as $r_f =]e,\infty[\subseteq d_g =]0,\infty[$

(c) $gof:]0,\infty[\mapsto \mathbb{R}$, where $gof(x) = (x+1) + \ln 2$

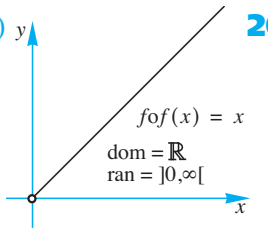


18. $(fog)(x) = |x|, x \in \mathbb{R}$; range $= [0,\infty[$

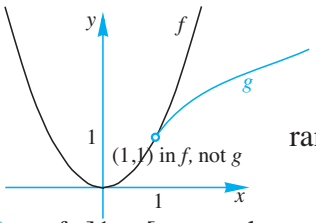
19. (a)



(c)



20. (a)



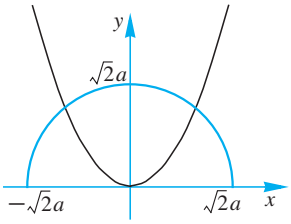
range $=]1,\infty[$

(b) $gof:]1,\infty[\mapsto \mathbb{R}$, where $gof(x) = x$

(d) $fog^*:]1,\infty[\mapsto \mathbb{R}$, where $gof(x) = x$

21. $d_f = \mathbb{R} \setminus \left\{\frac{a}{c}\right\}$, $r_f = \mathbb{R} \setminus \left\{\frac{a}{c}\right\}$, $r_f \subseteq d_f$, $fof(x) = x$

22. (a)



(b) $d_{fog} = [-\sqrt{2}a, \sqrt{2}a]$, $fog = 2a - \frac{x^2}{a}$

(c) $d_{gof} = [-2^{1/4}a, 2^{1/4}a]$, $fog = \frac{1}{a}\sqrt{2a^4 - x^4}$,

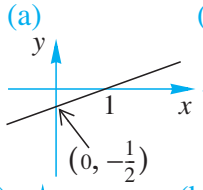
$range = [0, \sqrt{2}a]$

EXERCISE 5.4.2

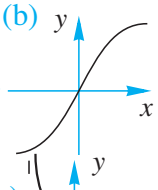
1. (a) $\frac{1}{2}(x-1), x \in \mathbb{R}$ (b) $\sqrt[3]{x}, x \in \mathbb{R}$ (c) $3(x+3), x \in \mathbb{R}$ (d) $\frac{5}{2}(x-2), x \in \mathbb{R}$ (e) $x^2 - 1, x > 0$

(f) $(x-1)^2, x \geq 1$ (g) $\frac{1}{x} - 1, x > 0$ (h) $\frac{1}{(x+1)^2}, x > -1$

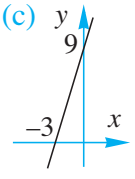
2. (a)



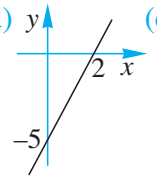
(b)



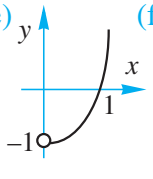
(c)



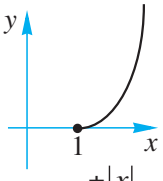
(d)



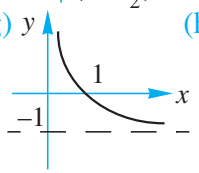
(e)



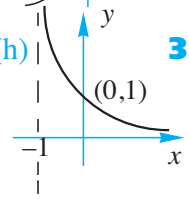
(f)



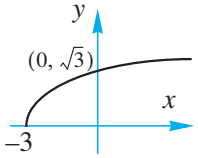
(g)

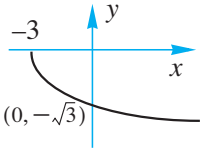


(h)

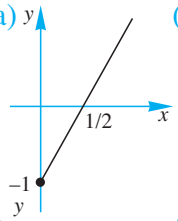


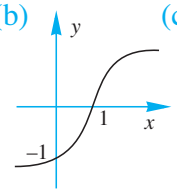
3. (a) $\sqrt{x+3}, x \geq -3$ (b) $-\sqrt{x+3}, x \geq -3$

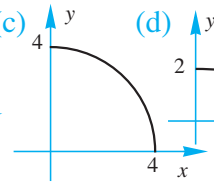


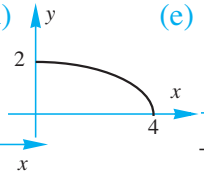


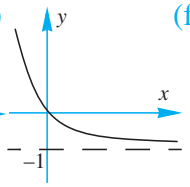
4. $\frac{\pm|x|}{\sqrt{1-x^2}}, -1 < x < 1$

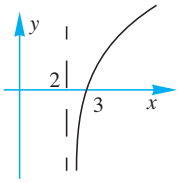
5. (a)


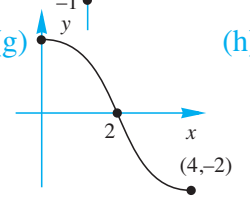
(b)


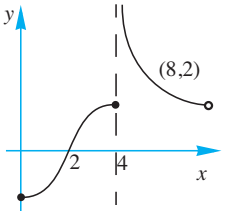
(c)


(d)


(e)


(f)


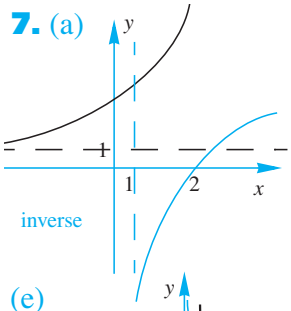
(g)


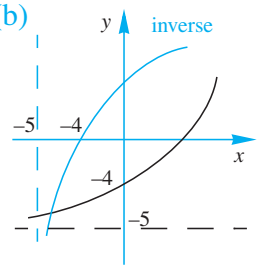
(h)


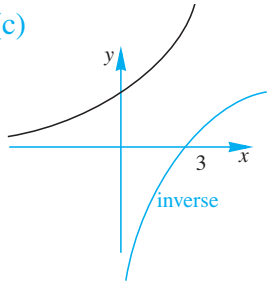
6. (a) $f^{-1}(x) = \log_3(x-1), x > 1$ (b) $f^{-1}(x) = \log_2(x+5), x > -5$

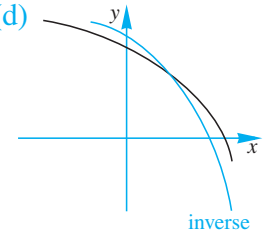
(c) $f^{-1}(x) = \frac{1}{2}(\log_3 x - 1), x > 0$ (d) $g^{-1}(x) = 1 + \log_{10}(3-x), x < 3$

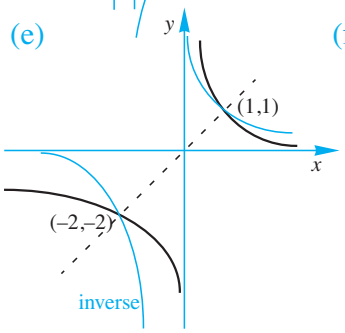
(e) $h^{-1}(x) = \log_3\left(1 + \frac{2}{x}\right), x \in \mathbb{R}\setminus[-2,0]$ (f) $g^{-1}(x) = \log_2\left(\frac{1}{x+1}\right), x > -1$

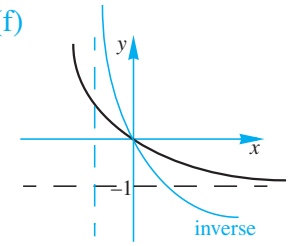
7. (a)


(b)


(c)


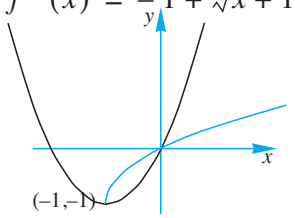
(d)


(e)


(f)


8. (a) $f^{-1}(x) = 2^x - 1, x \in \mathbb{R}$ (b) $f^{-1}(x) = \frac{1}{2} \cdot 10^x, x \in \mathbb{R}$ (c) $h^{-1}(x) = 2^{1-x}, x \in \mathbb{R}$

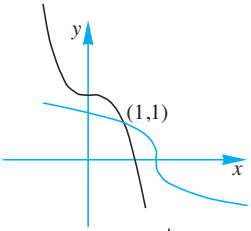
(d) $g^{-1}(x) = 3^{x+1} + 1, x \in \mathbb{R}$ (e) $h^{-1}(x) = 5^{x/2} + 5, x \in \mathbb{R}$ (f) $f^{-1}(x) = 1 - 10^{3(2-x)}, x \in \mathbb{R}$

9. $f^{-1}(x) = -1 + \sqrt{x+1}, x \geq -1$


dom = $[-1, \infty[$, ran = $[-1, \infty[$

10. (a) $f^{-1}(x) = a - x$ (b) $h^{-1}(x) = \frac{2}{x-a} + a$

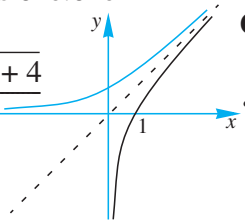
(c) $f^{-1}(x) = \sqrt{a^2 - x^2}$

11. $h^{-1}(x) = \sqrt[3]{2-x}$


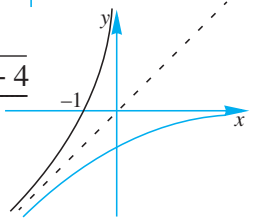
12. $[2, \infty[$ **13.** $\mathbb{R}^+\setminus\{1.5\}$

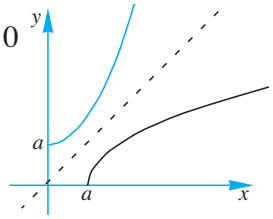
14. (a) Inverse exists as f is one:one

(b) **Case 1:** S = $]0, \infty[$

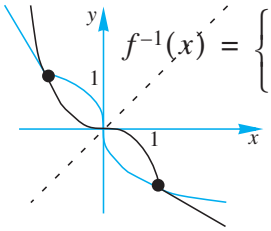
$g^{-1}(x) = \frac{x + \sqrt{x^2+4}}{2}$


Case 2: S = $]-\infty, 0[$

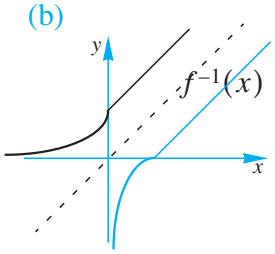
$g^{-1}(x) = \frac{x - \sqrt{x^2+4}}{2}$


15. $f^{-1}(x) = a(x^2+1), x \geq 0$ 

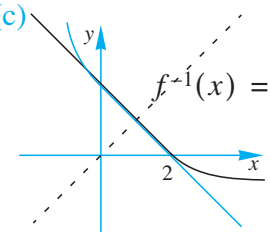
$\{x: f(x) = f^{-1}(x)\} = \varnothing$

16. (a) 

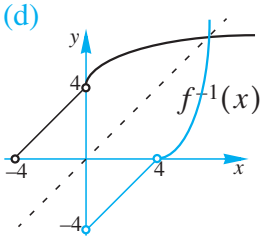
$f^{-1}(x) = \begin{cases} -(2x+1), & x < -1 \\ -\sqrt[3]{x}, & x \geq -1 \end{cases}$

(b) 

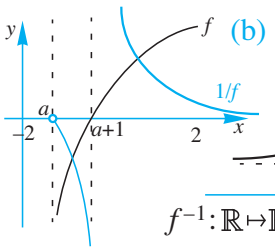
$f^{-1}(x) = \begin{cases} \ln(x) - 1, & 0 < x \leq e \\ x - e, & x > e \end{cases}$

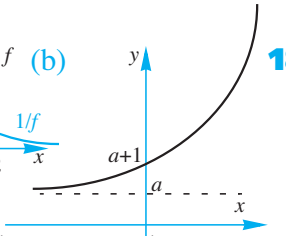
(c) 

$f^{-1}(x) = \begin{cases} 1 + e^{-x}, & x < 0 \\ 2 - x, & x \geq 0 \end{cases}$

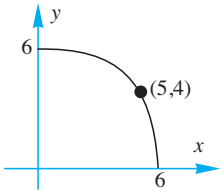
(d) 

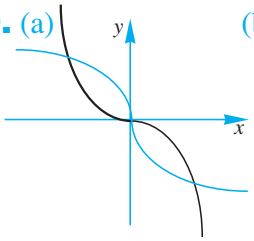
$f^{-1}(x) = \begin{cases} (x-4)^2, & x > 4 \\ x - 4, & 0 < x < 4 \end{cases}$

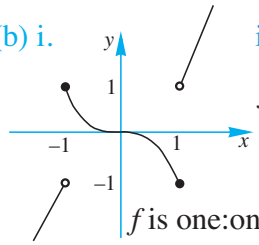
17. (a) 

(b) 

$f^{-1}: \mathbb{R} \mapsto \mathbb{R}$, $f^{-1}(x) = a + e^{ax}$

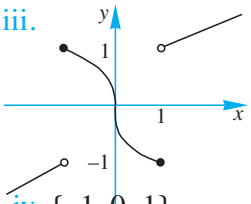
18. gof exists as $r_f \subseteq d_g$. It is one:one so the inverse exists:


19. (a) 

(b) i. 

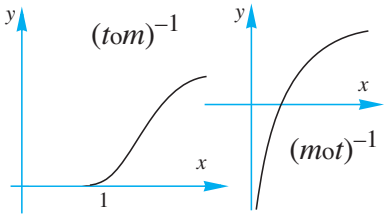
f is one:one

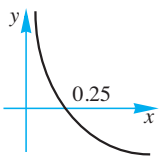
ii. $f(x) = \begin{cases} \frac{1}{2}(x-1) & x < -1 \\ -\sqrt[3]{x} & -1 \leq x \leq 1 \\ \frac{1}{2}(x+1) & x > 1 \end{cases}$

iii. 

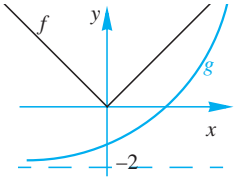
iv. $\{-1, 0, 1\}$

20. (a) i. $tom(x) = e^{\sqrt{x}}, x \geq 0$ ii. $mot(x) = \sqrt{e^x}, x \in \mathbb{R}$ (b) i. $(tom)^{-1}(x) = (\ln(x))^2, x > 1$
ii. $(mot)^{-1}(x) = \ln x^2, x > 0$ (c) i. & ii. neither exist
(d) Adjusting domains so that the functions in (c) exist, we have: $t^{-1}om^{-1}(x) = (mot)^{-1}(x)$ & $m^{-1}ot^{-1}(x) = (tom)^{-1}(x)$
(e) Yes as rules of composition OK.

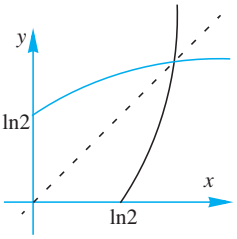


21. (a) 1 (b) 

0.206

22. (a) 

(b) fog exists but is not one:one

(c) i. B = [ln2, ∞[
ii. $(fog)^{-1}$: [0,∞[$\mapsto \mathbb{R}$ where, $(fog)^{-1}(x) = \ln(x+2)$ iii. 

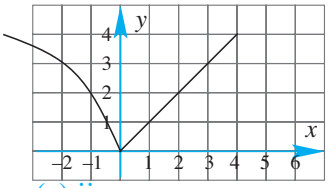
EXERCISE 6.1

1. (a) $y = (x-4)^2$ (b) $y = (x+2)^2$ (c) $y = x^2 + 5$ (d) $(x-2)^2 + y = 2$ (e) $x^2 + y = 4$

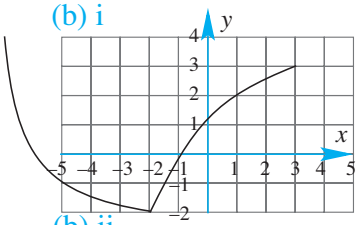
(f) $x^2 + y = 0$ (g) $y = \dfrac{8}{x-4}, x \neq 4$ (h) $y = \dfrac{8}{x} - 1, x \neq 0$ (i) $(x+1)^2 + y^2 = 4$

(j) $y^2 = \dfrac{9}{x-3}, x \neq 3$ (k) $(y+3)^2 = \dfrac{9}{x}, x \neq 0$ (l) $x + y^2 = 8$

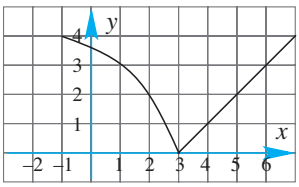
2. (a) i



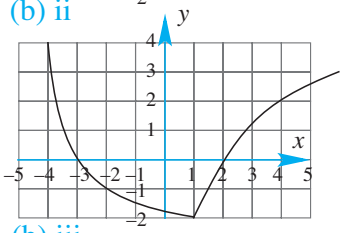
(b) i



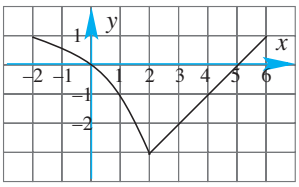
(a) ii



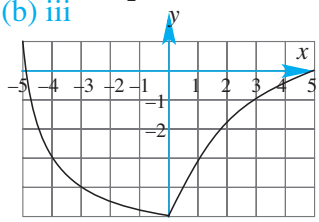
(b) ii



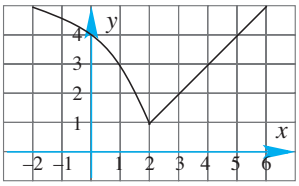
(a) iii



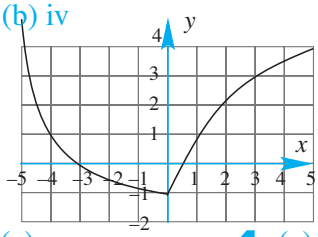
(b) iii



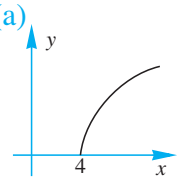
(a) iv



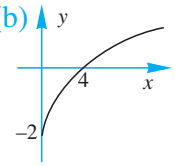
(b) iv



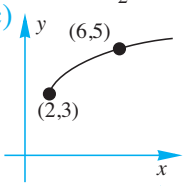
3. (a)



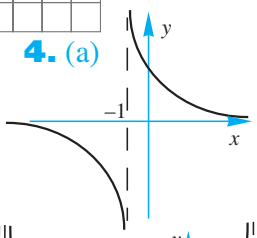
(b)



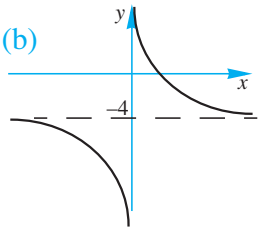
(c)



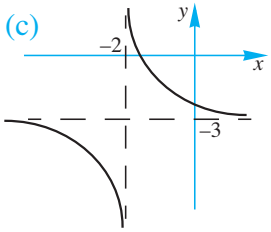
4. (a)



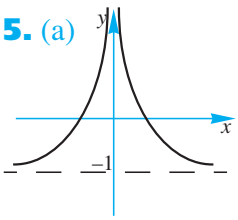
(b)



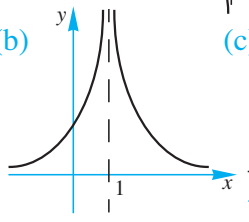
(c)



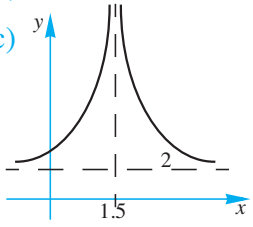
5. (a)



(b)

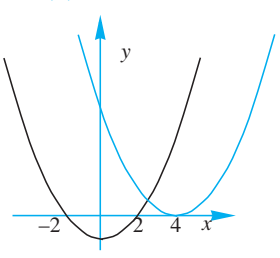


(c)



6. First function in black, second function in blue

(a)



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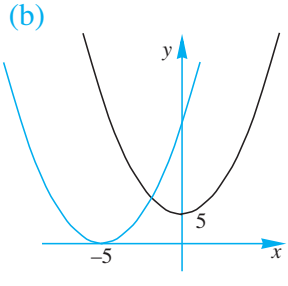
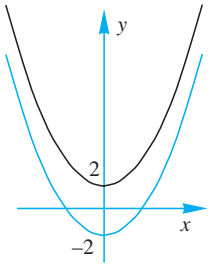
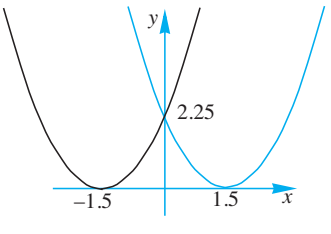
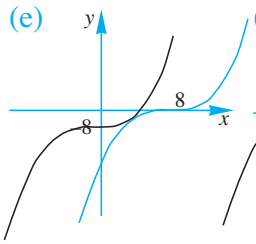


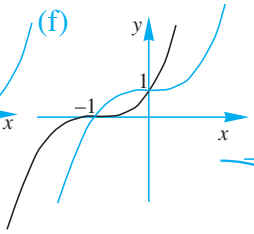
(c)

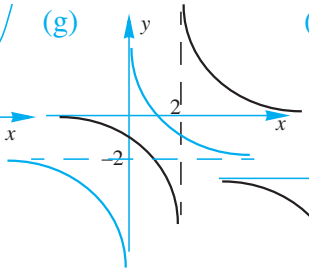


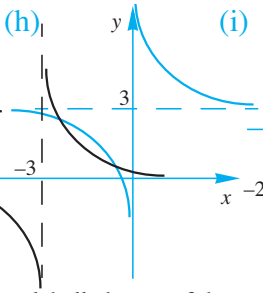
(d)

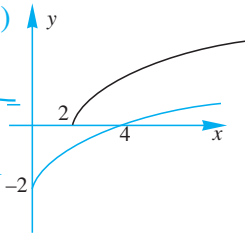


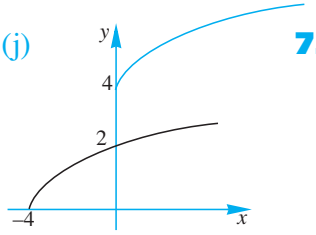
(e) 

(f) 

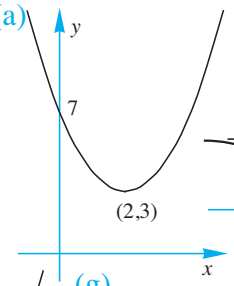
(g) 

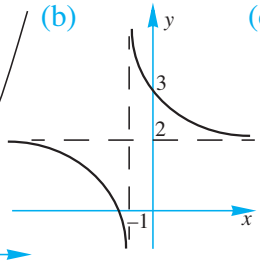
(h) 

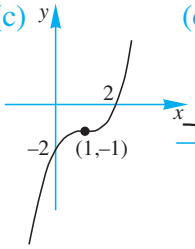
(i) 

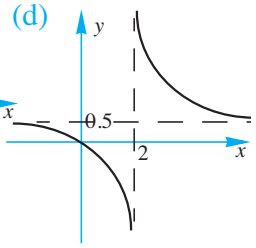
(j) 

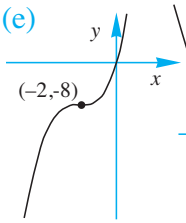
7. Note:coordinates were asked for. We have labelled most of these with single numbers.

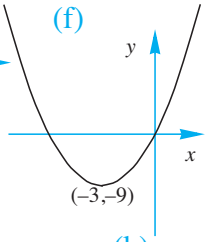
(a) 

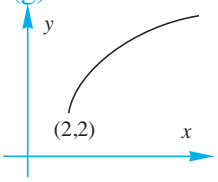
(b) 

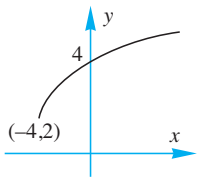
(c) 

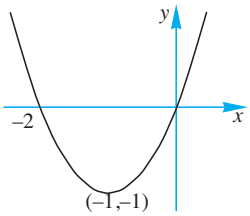
(d) 

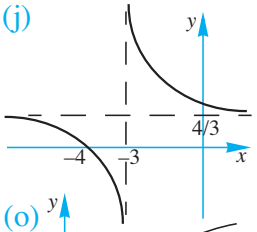
(e) 

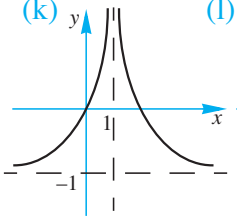
(f) 

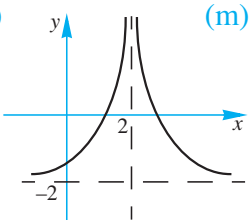
(g) 

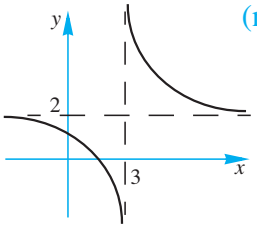
(h) 

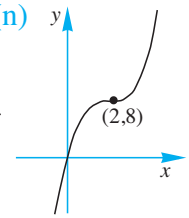
(i) 

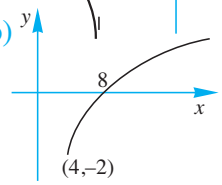
(j) 

(k) 

(l) 

(m) 

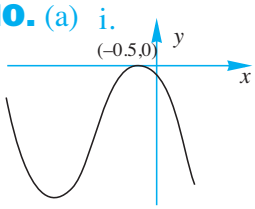
(n) 

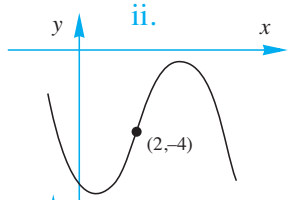
(o) 

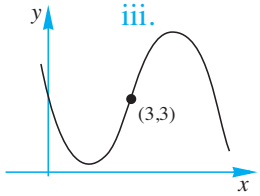
8. (a) $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$ (b) $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ (c) $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$ (d) $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ (e) $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ (f) $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$ (g) $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$ (h) $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$

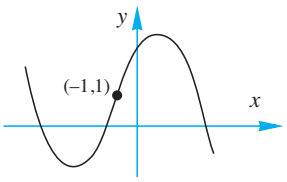
(i) $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$ (j) $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ (k) $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ (l) $\begin{pmatrix} -k \\ h \end{pmatrix}$ (m) $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$ (n) $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ (o) $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$

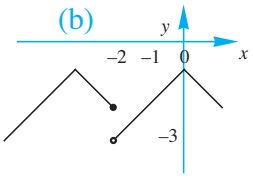
9. (a) $g(x) = f(x-1)+1$ (b) $g(x) = f(x+2)-4$ (c) $g(x) = f(x-2)$
(d) $g(x) = f(x-1)+1$ (e) $g(x) = f(x-1)+3$

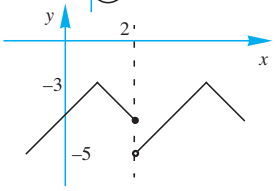
10. (a) i. 

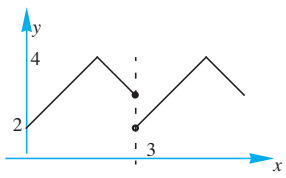
ii. 

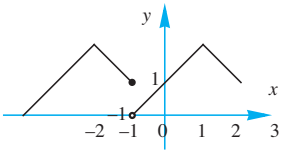
iii. 

iv. 

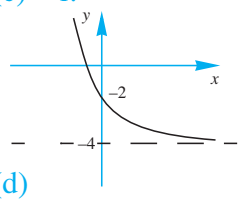
(b) 



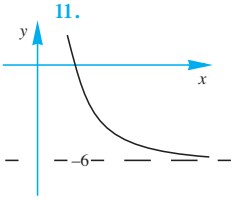




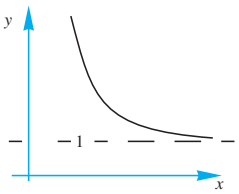
(c) i.



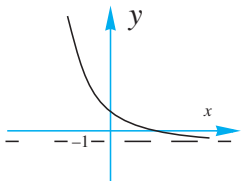
ii.



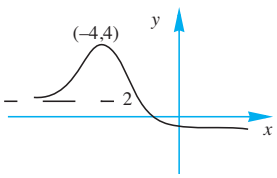
iii.

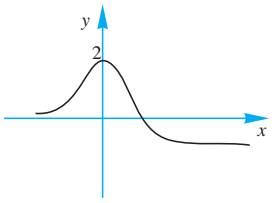


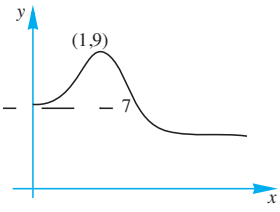
iv.

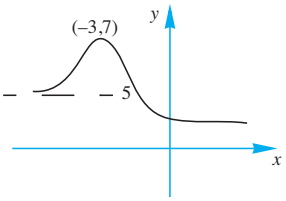


(d)





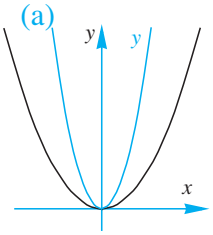




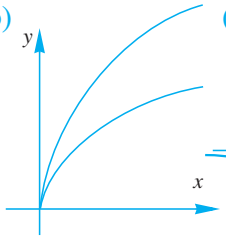
11. $y = \begin{cases} f(x+2)+2, -3 \le x \le -1 \\ f(x+4)+2, -5 \le x \le -3 \end{cases}$

EXERCISE 6.2

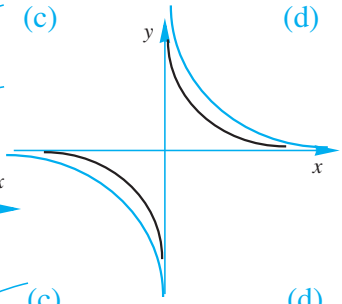
1. (a)



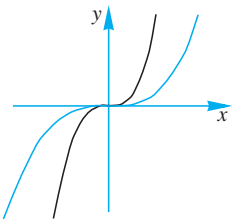
(b)



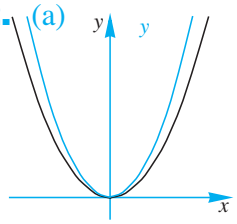
(c)



(d)



2. (a)



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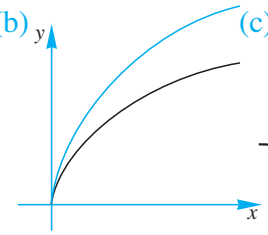

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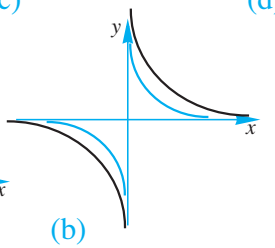

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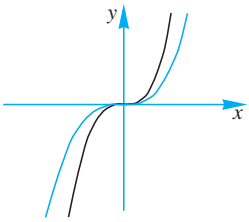
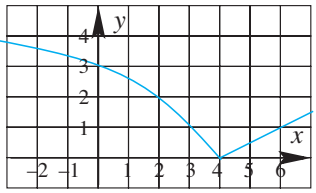
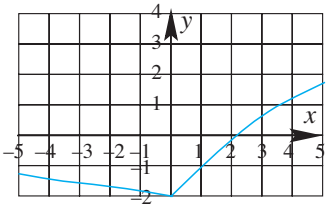


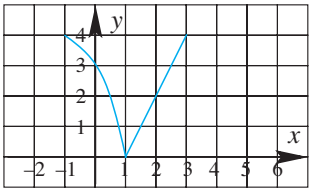
3. (a) (b)

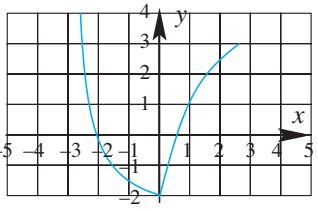
i.



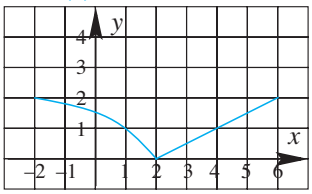


ii.

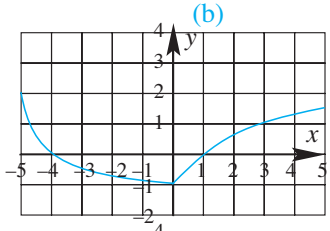




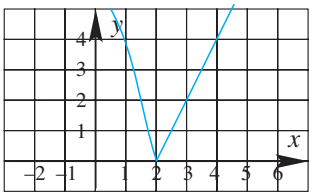
iii. (a)

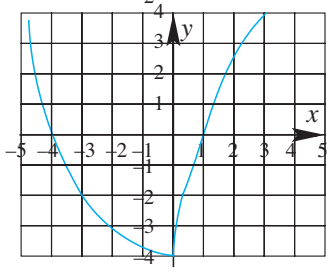


(b)



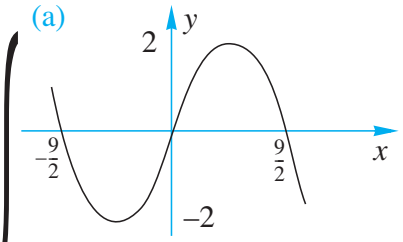
iv.



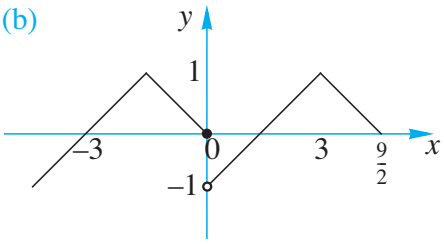


4. i. $y = f\left(\frac{2}{3}x\right)$

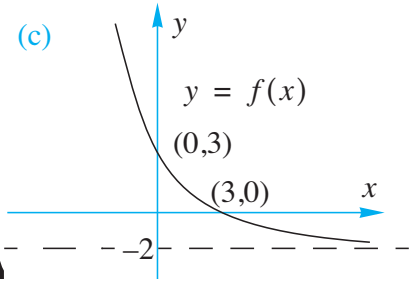
(a)



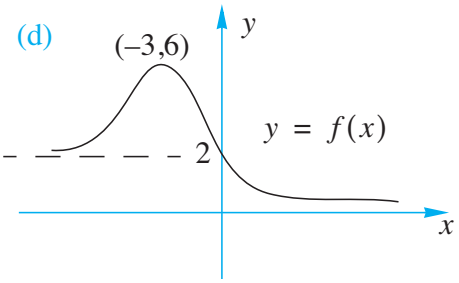
(b)



(c)

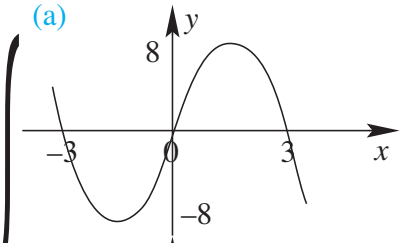


(d)

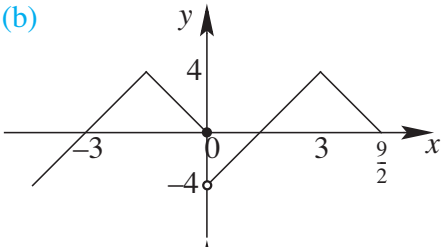


ii. $y = 4f(x)$

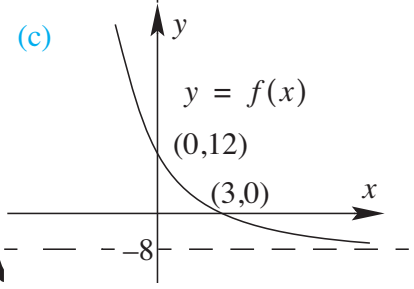
(a)



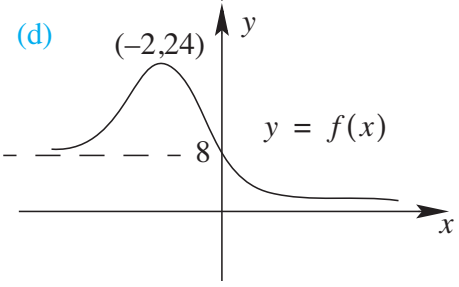
(b)



(c)



(d)

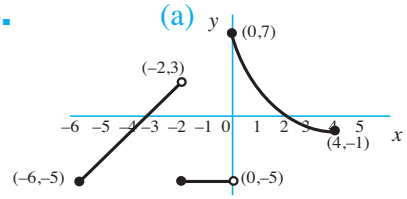


5. (a) $f(x) = |x| \quad y = f(2x) + 1$ (b) $f(x) = x^2 \quad y = \frac{1}{2}f(x-2) - 3$

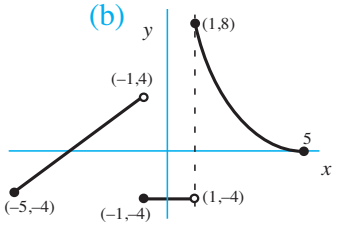
(c) $f(x) = \frac{1}{x} \quad y = \frac{1}{2}f\left(x - \frac{1}{2}\right)$ (d) $f(x) = x^3 \quad y = 27f\left(x - \frac{2}{3}\right)$

(e) $f(x) = x^4 \quad y = 128f\left(x - \frac{1}{2}\right) - 2$ (f) $f(x) = \sqrt{x} \quad y = \sqrt{2}f(x) + 2$

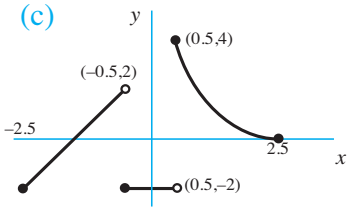
6. (a)



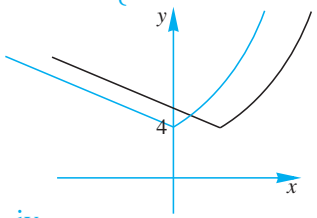
(b)



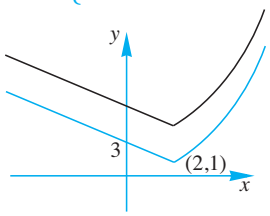
(c)



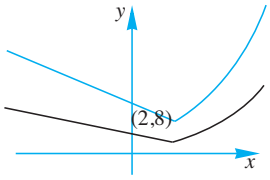
7. i.
$$f(x) = \begin{cases} (x+2)^2 & \text{if } x \geq 0 \\ 4 - x & \text{if } x < 0 \end{cases}$$



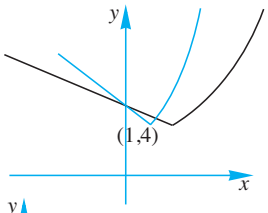
ii.
$$h(x) = \begin{cases} x^2 - 3 & \text{if } x \geq 2 \\ 3 - x & \text{if } x < 2 \end{cases}$$



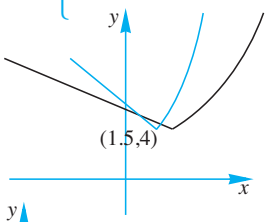
iii.
$$h(x) = \begin{cases} 2x^2 & \text{if } x \geq 2 \\ 12 - 2x & \text{if } x < 2 \end{cases}$$



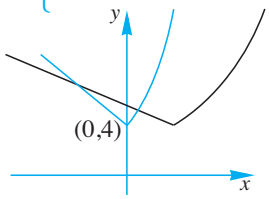
iv.
$$k(x) = \begin{cases} 4x^2 & \text{if } x \geq 1 \\ 6 - 2x & \text{if } x < 1 \end{cases}$$



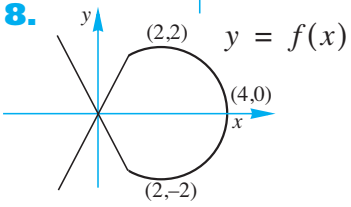
v.
$$k(x) = \begin{cases} (2x-1)^2 & \text{if } x \geq \frac{3}{2} \\ 7 - 2x & \text{if } x < \frac{3}{2} \end{cases}$$



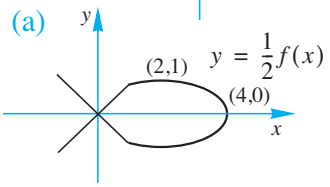
vi.
$$f(x) = \begin{cases} \frac{1}{2}(4x+2)^2 & \text{if } x \geq 0 \\ 2 - 2x & \text{if } x < 0 \end{cases}$$



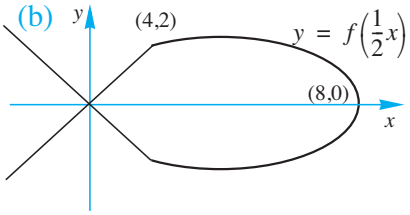
8.



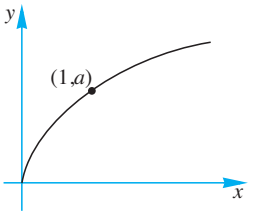
(a)



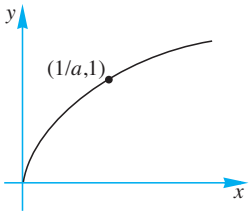
(b)



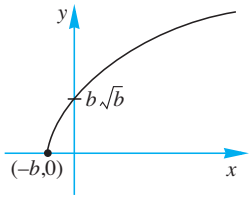
9. (a)



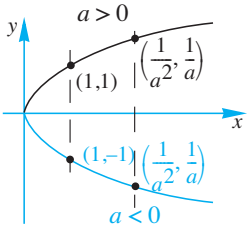
(b)



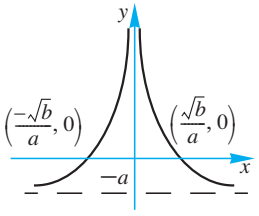
(c)



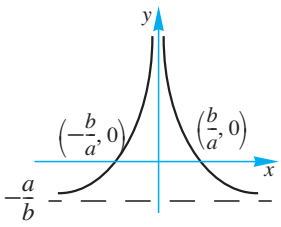
(d)



10. (a)



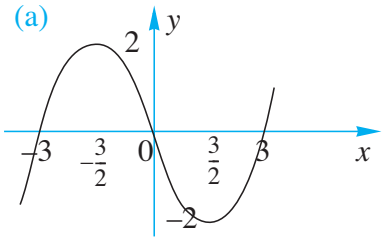
(b)



EXERCISE 6.3

1. i.

(a)



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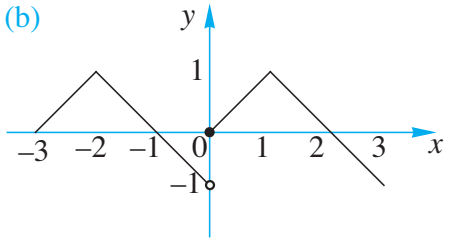
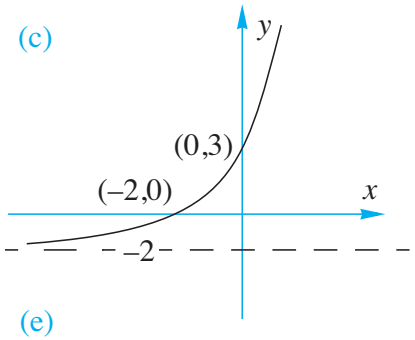
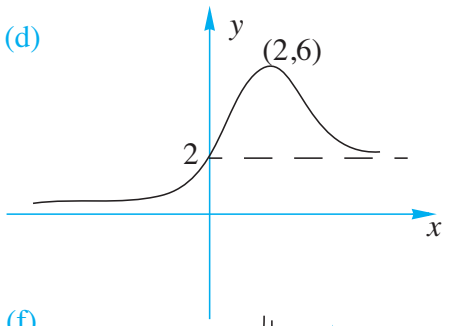


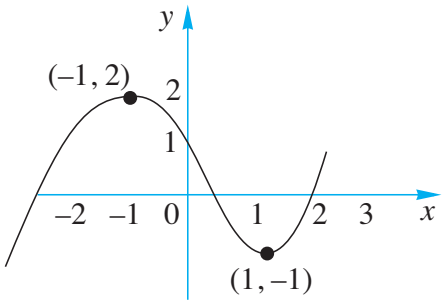
(c)



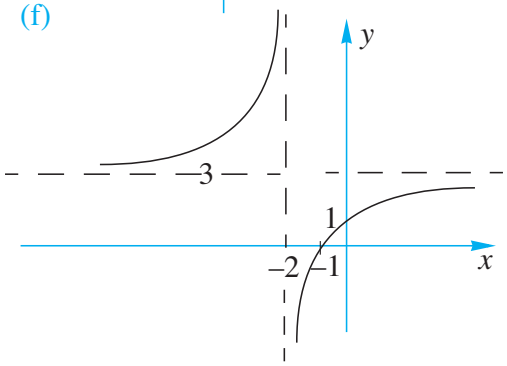
(d)



(e)

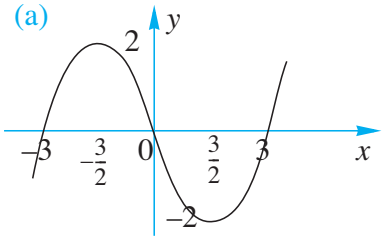


(f)

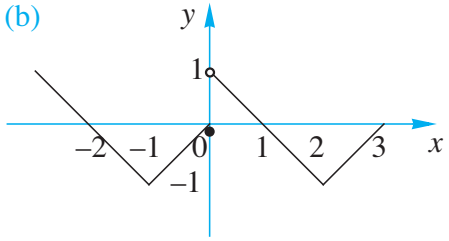


ii.

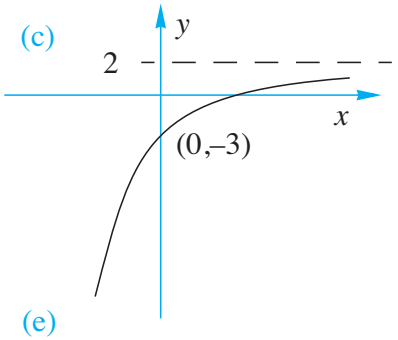
(a)



(b)



(c)



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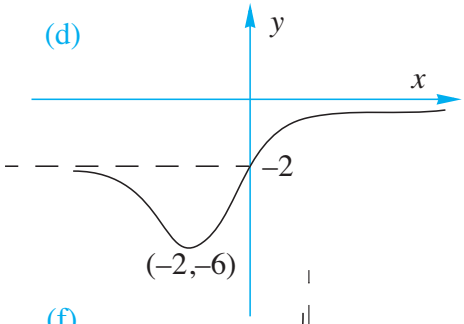

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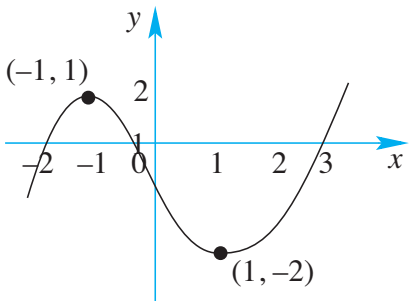

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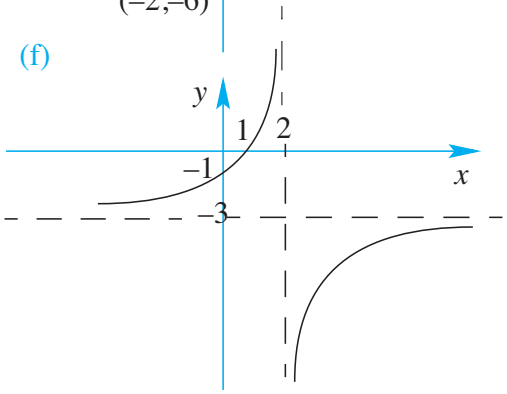
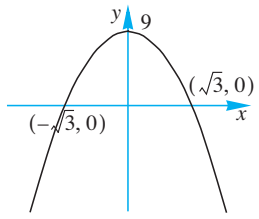


2. (a) $y = -f(x)$ (b) $y = f(-x)$ (c) $y = f(x+1)$ (d) $y = f(2x)$ (e) $y = 2f(x)$

3. (a)



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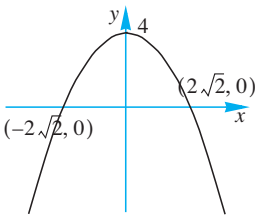

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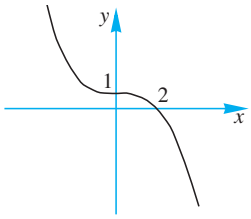

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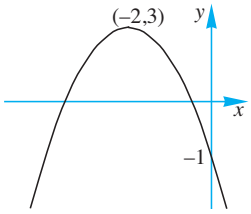

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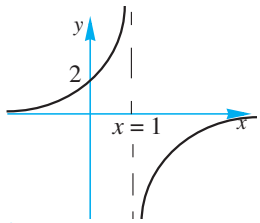

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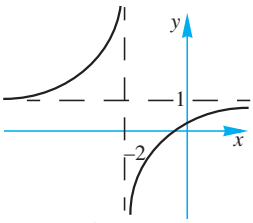

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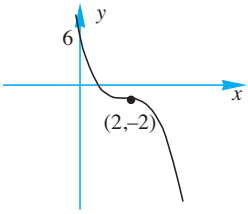

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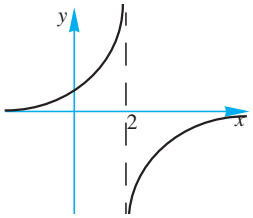

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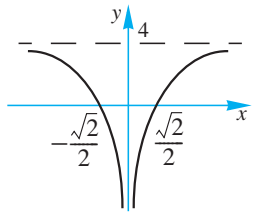

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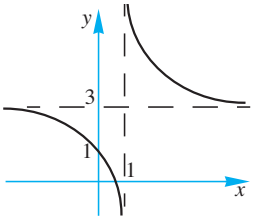

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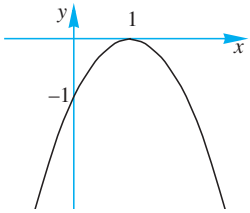

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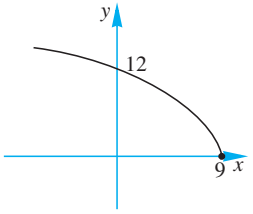

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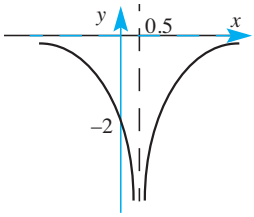

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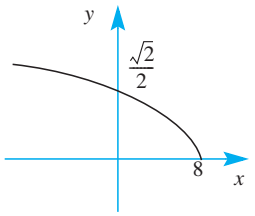

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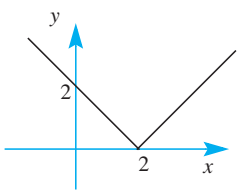

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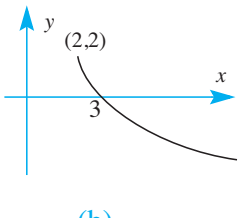

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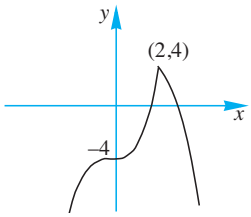

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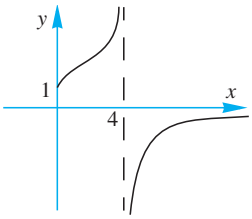

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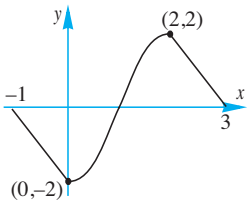

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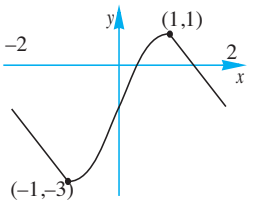

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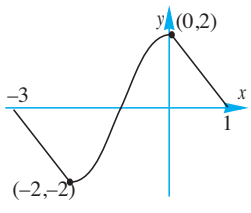

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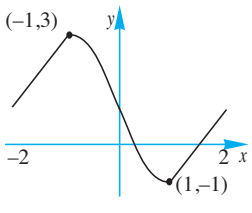

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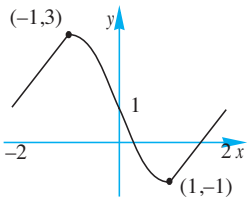

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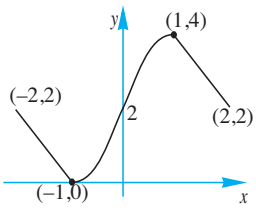

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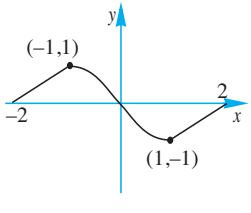

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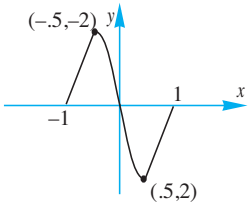

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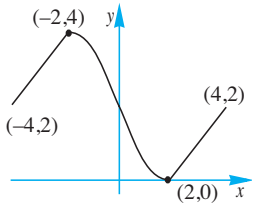

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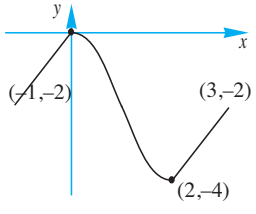

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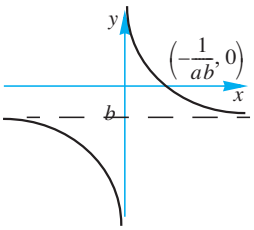
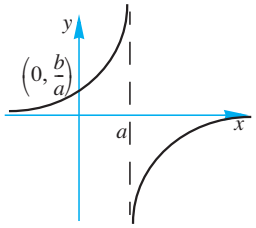
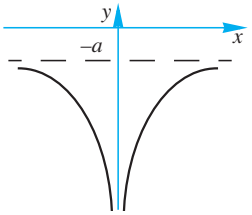


(a) ii



(b) i



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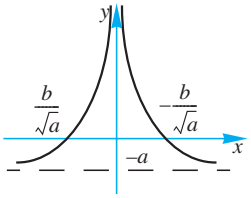
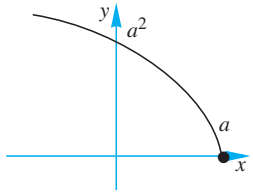


(c) i



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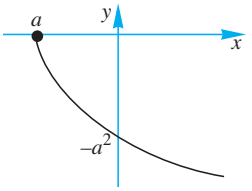
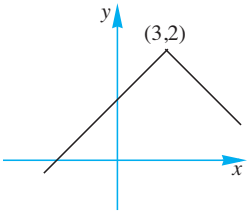


6. (a)



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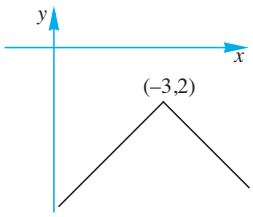

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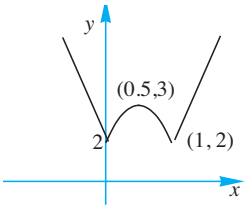

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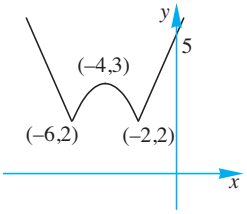

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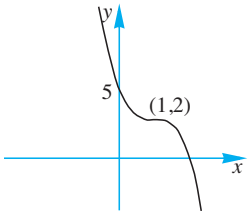

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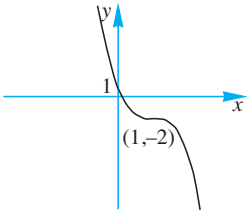
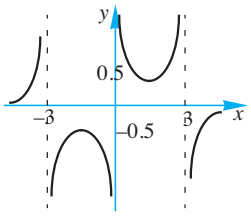


EXERCISE 6.4

1. (a)



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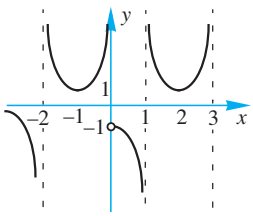

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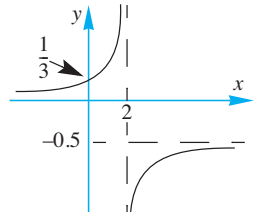

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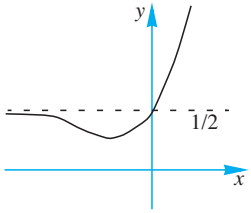

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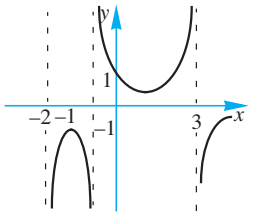

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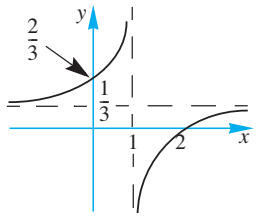
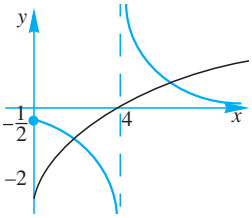


2. (a)



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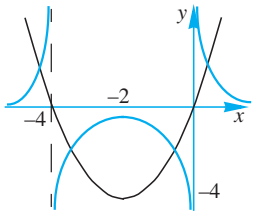

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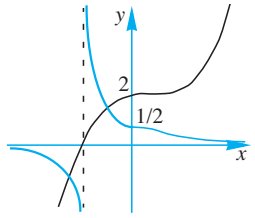

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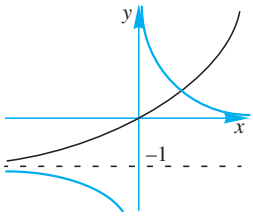

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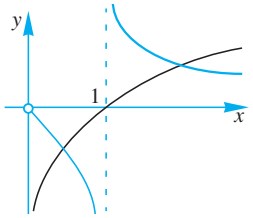

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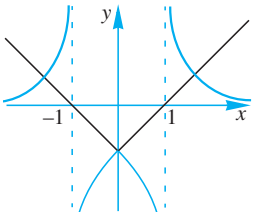

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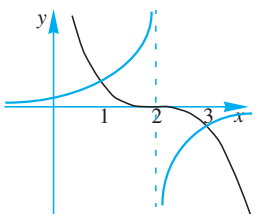

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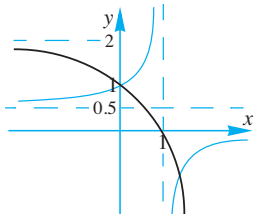
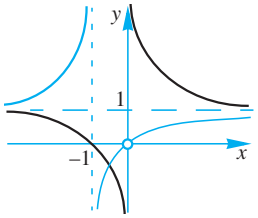


(i)



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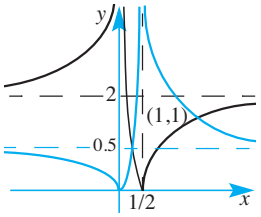

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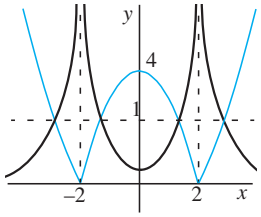

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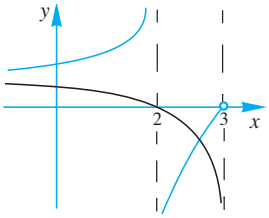
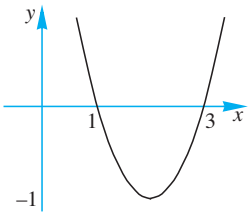
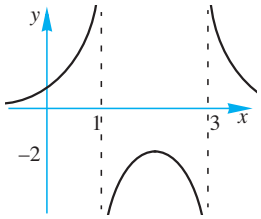


3. (a)



(b)



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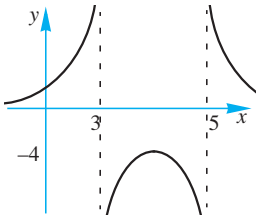
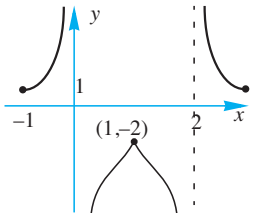


EXERCISE 6.5

1. (a) i



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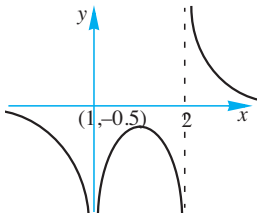

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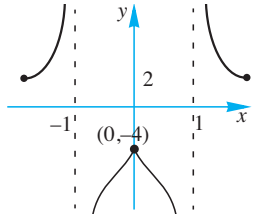

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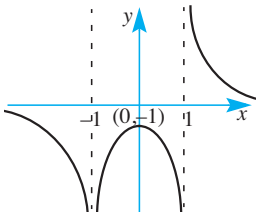
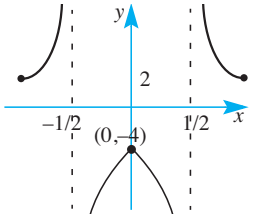


(c) i



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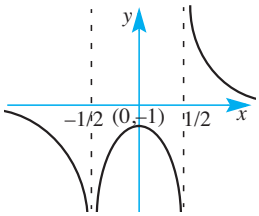

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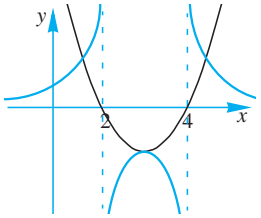

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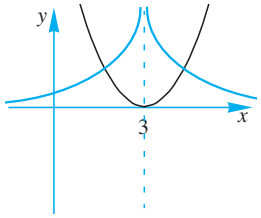

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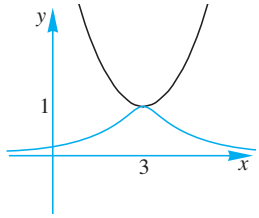
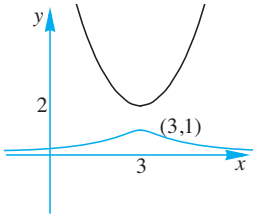


(d)



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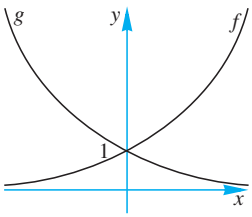

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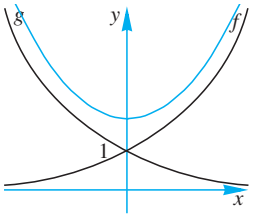

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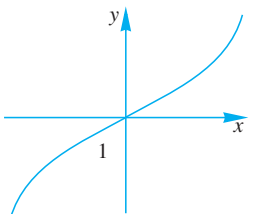
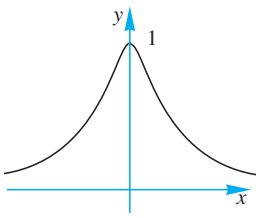
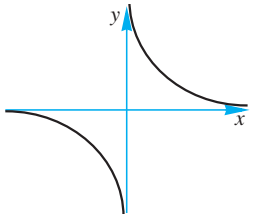
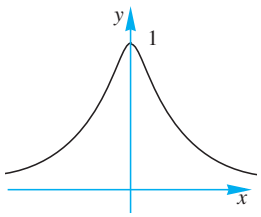
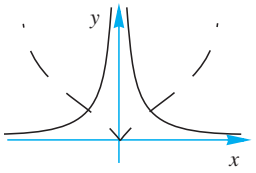
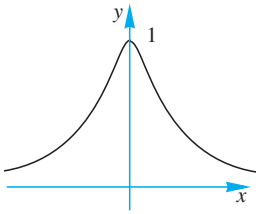


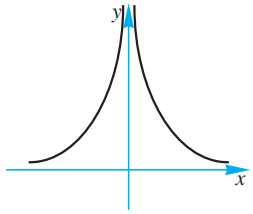
(c) i 

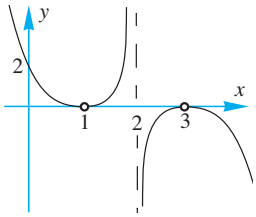
(c) ii 

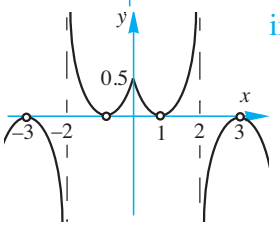
(c) iii 

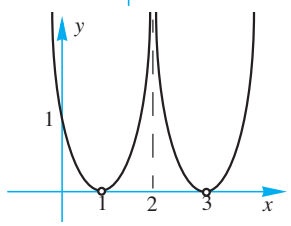
(c) iv 

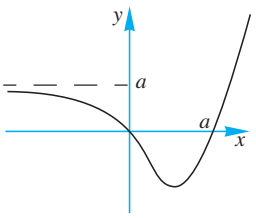
(c) v 

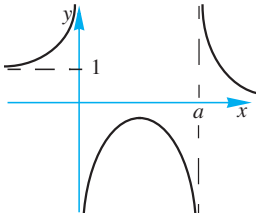
(c) vi 

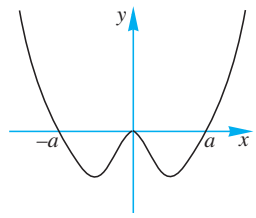
4. i 

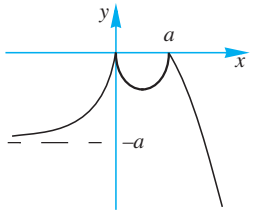
ii 

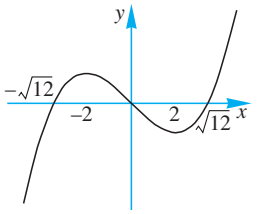
iii 

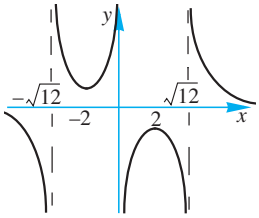
5. (a) 

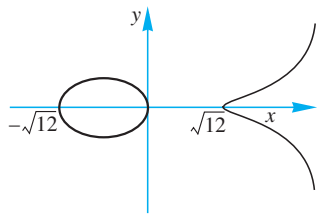
(b) 

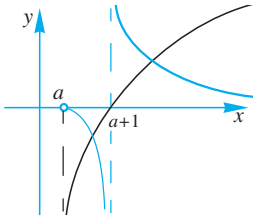
(c) i 

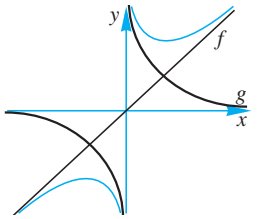
(c) ii 

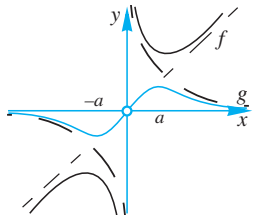
6. (a) 

(b) i 

(b) ii 

7. 

8.(a) 

(b) 

EXERCISE 7.1.1

1. (a) $\frac{27y^{15}}{8x^3}$ (b) $\frac{91}{216a^6}$ (c) $2^n + 2$ (d) $\frac{8x^{11}}{27y^2}$ (e) $\frac{3x^2y^2}{8}$ (f) $3^{n+1} + 3$ (g) $4^{n+1} - 4$

(h) $2(4^{n+1} - 4)$ (i) $\frac{1-b^6}{16b^4}$ **2.** (a) 64 (b) $\left(\frac{2}{3}\right)^x$ (c) 2^{2y+1} (d) $\frac{1}{b^{2x}}$ (e) $\left(\frac{y}{2}\right)^6$ (f) $\left(\frac{9}{2}\right)^{n+2}$ **3.** (a) $\frac{z^2}{xy}$

(b) 3^{7n-2} (c) 5^{n+1} (d) 9 (e) 2^{6n+1} (f) 2^{1-3n} (g) x^{2+4n-n^2} (h) x^{3n^2+n+1} (i) 27 **4.** $\frac{y^{2m-2}}{x^m}$

5. (a) –81 (b) $-\frac{9x^8}{8y^4}$ (c) $y - x$ (d) $\frac{2x+1}{x+1}$ (e) –1 (f) $-b$ **6.** (a) $\frac{1}{x^2y^2}$ (b) $\frac{1}{x^4}$ (c) $-\frac{1}{x(x+h)}$

(d) $\frac{1}{x-1}$ (e) $\frac{1}{(x+1)(x-1)^5}$ (f) $\frac{1}{x^2}$ **7.** (a) $118 \times 5^{n-2}$ (b) 1 (c) $\frac{b^7}{a^4}$ (d) a^{mn} (e) $\frac{p+q}{pq}$ (f) $\frac{2\sqrt{a}}{a-1}$

(g) $\frac{7}{8}$ (h) $a^{7/8}$ **8.** (a) $x^{11/12}$ (b) $2a^{3n-2}b^{2n-2}$ (c) 2^n (d) $-\frac{7^{m-n}}{8}$ (e) $\frac{6 \times 5^n}{5^n+5}$ (f) $x+1$

EXERCISE 7.1.2

1. (a) 2 (b) –2 (c) $\frac{2}{3}$ (d) 5 (e) 6 (f) –2.5 (g) 2 (h) 1.25 (i) $\frac{1}{3}$ **2.** (a) –6 (b) $-\frac{2}{3}$ (c) –3 (d) 1.5 (e) 0.25

(f) 0.25 (g) $-\frac{1}{8}$ (h) $-\frac{11}{4}$ (i) –1.25

EXERCISE 7.1.3

1. (a) 3.5 (b) 3.5 (c) –3 (d) 1.5 (e) 3.5 (f) 1.5 (g) 1.8 (h) $-\frac{4}{7}$ (i) 0 **2.** (a) –0.75 (b) –1,4 (c) 0,1

(d) 3,4 (e) –1,4 (f) 0,2 **3.** (a) –1,1,2 (b) –3,1, 3,4 (c) $\frac{4}{3}, \frac{5}{3}$, 2 (d) –1,1,2 (e) 3,7, $\frac{-1 \pm \sqrt{233}}{2}$, $\frac{1}{3}$

EXERCISE 7.1.4

1. (a) i. 5.32 ii. 9.99 iii. 2.58 (b) i. 2.26 ii. 3.99 iii. 5.66 (c) i. 3.32 ii. –4.32 iii. –6.32 (d) i. –1.43
ii. 1.68 iii. –2.86 **2.** (a) 0 (b) 0.54 (c) –0.21 (d) –0.75, 0 (e) 1.13 (f) 0, 0.16

EXERCISE 7.1.5

1. (a) 2 (b) –1 (c) 0.5 (d) 0.5 **2.** (a) 1 (b) 0.6 (c) 0 **3.** (a) 0 (b) $\frac{2}{3}$ **4.** (a) –1,2 (b) –2,3 (c) –1

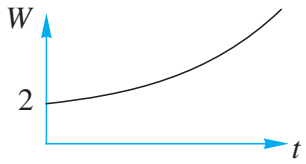
(d) –6,1 (e) 0,1 (f) 1 **5.** (a) 1.3863 (b) 2.1972 (c) 3.2189 (d) Ø **6.** (a) 0.4236 (b) 0.4055

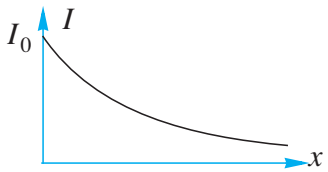
(c) 0.3054 (d) –0.4176 **7.** (a) 0 (b) –0.6733 (c) 0 **9.** 36 **10.** $a = \sqrt{2}e, k = \ln(\sqrt{2})$

EXERCISE 7.2

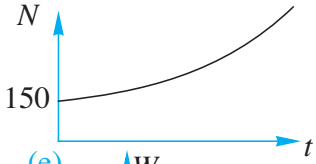
1. (a) 1000 (b) 1516 (c) 2000 (d) 10 days **2.** (a) 0.0013 (b) 2.061 kg (c) 231.56 yrs

(d) **3.** (a) 0.01398 (b) 52.53% (c) 51.53m (d) 21.53m

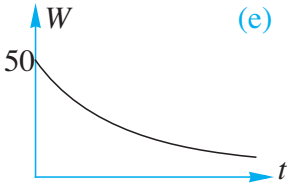




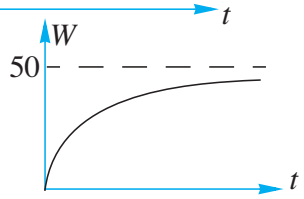
4. (a) i. 157 ii. 165 iii. 191 (b) 14.2 yrs (c) 20.1 yrs (d)



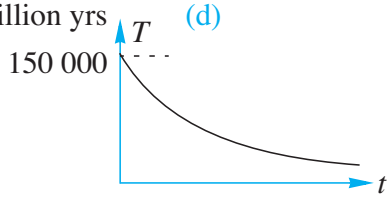
5. (a) 50 (b) 0.0222 (c) 17.99 kg (d)



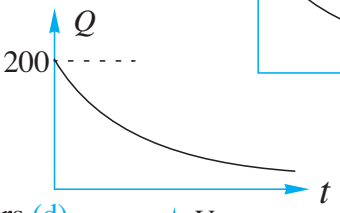
(e)



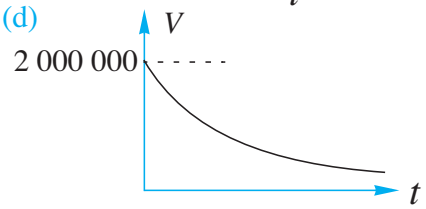
6. (a) 15 000˚C (b) i. 11 900˚C ii. 1500˚C (c) 3.01 million yrs (d)



7. (a) 0.0151 (b) 12.50gm (c) 20 years (d)



8. (a) \$2 million (b) \$1.589 mil (c) 30.1 years (d)

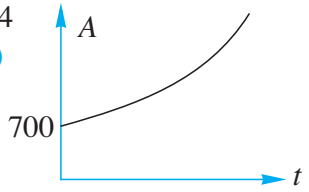


9. (b) 0.01761 (c) 199 230 (d) 22.6 years

10. (a) 20 cm^2 (b) 19.72 cm^2 (c) 100 days (d) 332 days

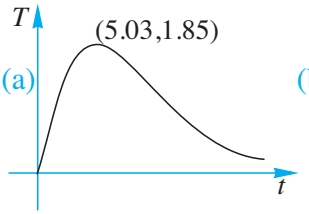
11. (a) 1 (b) i. 512170 ii. 517217 (c) 54.1 early 2014

12. (a) i. \$933.55 ii. \$935.50 (b) 11.95 years (c)



13. (a) 99 (b) $99 \times 2^{0.1394t}$ (c) 684

14. (a)

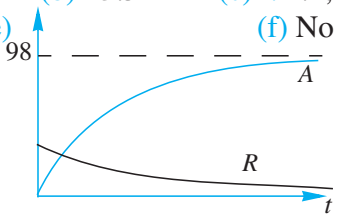


(b) 38.85˚C at ~ midnight

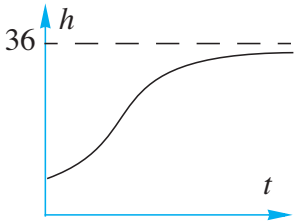
15. (a) 19 (b) 2.63 (c) 100

16. (a) 18 cm (b) 4 cm (c) 1.28 m (d) 36 m (e) i. 21.7yr ii. 27.6yr iii. 34.5yr (f) 36 (g)

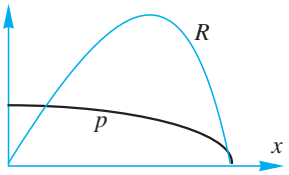
17. (a) 5 mg/min (b) 13.51 min (c) i. 2.1, ii. 13.9 iii. 68min (d) 19.6 mg (e)



(f) No



18. (a) i. \$499 ii. \$496 iii. \$467 (c) 15537 (d) i. \$499k ii. \$2.48mil iii. \$4.67mil (f) 12358 (g) \$5.14mil (b), (e)



EXERCISE 7.3

1. (a) 2 (b) 2 (c) 5 (d) 3 (e) –3 (f) –2 (g) 0 (h) 0 (i) –1 (j) –2 (k) 0.5 (l) –2 **2.** (a) $\log_{10}10000 = 4$ (b) $\log_{10}0.001 = -3$ (c) $\log_{10}(x+1) = y$ (d) $\log_{10}p = 7$ (e) $\log_2(x-1) = y$ (f) $\log_2(y-2) = 4x$ **3.** (a) $2^9 = x$ (b) $b^x = y$ (c) $b^{ax} = t$ (d) $10^{x^2} = z$ (e) $10^{1-x} = y$ (f) $2^y = ax - b$ **4.** (a) 16 (b) 2 (c) 2 (d) 9 (e) $\sqrt[4]{2}$ (f) 125 (g) 4 (h) 9 (i) $\sqrt[3]{\frac{1}{3}}$ (j) 21 (k) 3 (l) 13

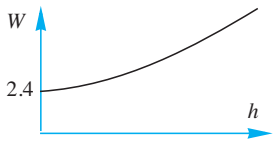
5. (a) 54.5982 (b) 1.3863 (c) 1.6487 (d) 7.3891 (e) 1.6487 (f) 0.3679 (g) 52.5982 (h) 4.7183 (i) 0.6065

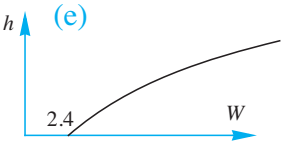
EXERCISE 7.4

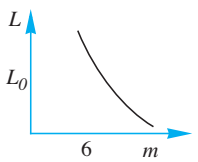
1. (a) 5 (b) 2 (c) 2 (d) 1 (e) 2 (f) 1 **2.** (a) $\log a = \log b + \log c$ (b) $\log a = 2\log b + \log c$
(c) $\log a = -2\log c$ (d) $\log a = \log b + 0.5\log c$ (e) $\log a = 3\log b + 4\log c$
(f) $\log a = 2\log b - 0.5\log c$ **3.** (a) 0.18 (b) 0.045 (c) –0.09 **4.** (a) $x = yz$ (b) $y = x^2$
(c) $y = \dfrac{x+1}{x}$ (d) $x = 2^{y+1}$ (e) $y = \sqrt{x}$ (f) $y^2 = (x+1)^3$ **5.** (a) $\dfrac{1}{2}$ (b) $\dfrac{1}{2}$ (c) $\dfrac{17}{15}$ (d) $\dfrac{3}{2}$ (e) $\dfrac{1}{3}$
(f) no real sol'n (g) 3,7 (h) $\dfrac{\sqrt{33}-1}{2}$ (i) 4 (j) $\sqrt{10}+3$ (k) $\dfrac{64}{63}$ (l) $\dfrac{2}{15}$ **6.** (a) $\log_3 2wx$ (b) $\log \dfrac{x}{47y}$
(c) $\log_a[x^2(x+1)^3]$ (d) $\log_a\left[\dfrac{(x^5)(x+1)^3}{\sqrt{2x-3}}\right]$ (e) $\log_{10}\left(\dfrac{y^2}{x}\right)$ (f) $\log_2\left(\dfrac{y}{x}\right)$ **7.** (a) 1 (b) –2 (c) 3 (d) 9
(e) 2 (f) 9 **8.** (a) 1,4 (b) $1,3^{\pm\sqrt{3}}$ (c) $1,4^{\sqrt[3]{4}}$ (d) $1,5^{\pm\sqrt[4]{5}}$ **9.** (a) $\dfrac{\log 14}{\log 2} = 3.81$ (b) $\dfrac{\log 8}{\log 10} = 0.90$
(c) $\dfrac{\log 125}{\log 3} = 4.39$ (d) $\dfrac{1}{\log 2} \times \log\left(\dfrac{11}{3}\right) - 2 = -0.13$ (e) $\dfrac{\log 10 - \log 3}{4\log 3} = 0.27$ (f) 5.11
(g) $\dfrac{-\log 2}{2\log 10} = -0.15$ (h) 7.37 (i) 0.93 (j) no real solution (k) $\dfrac{\log 3}{\log 2} - 2 = -0.42$
(l) $\dfrac{\log 1.5}{\log 3} = 0.37$ **10.** (a) 0.5,4 (b) 3 (c) –1,4 (d) $10,10^{10}$ (e) 5 (f) 3 **11.** (a) $(4, \log_4 11)$
(b) (100,10) (c) (2,1) **12.** (a) $y = xz$ (b) $y = x^3$ (c) $x = e^{y-1}$ **13.** (a) $\dfrac{1}{e^4-1}$ (b) $\dfrac{1}{3}$ (c) $\dfrac{\sqrt{5}-1}{2}$
(d) Ø **14.** (a) $\ln 21 = 3.0445$ (b) $\ln 10 = 2.3026$ (c) $-\ln 7 = -1.9459$ (d) $\ln 2 = 0.6931$
(e) $\ln 3 = 1.0986$ (f) $2\ln\left(\dfrac{14}{9}\right) = 0.8837$ (g) $e^3 = 20.0855$ (h) $\dfrac{1}{3}e^2 = 2.4630$
(i) $\pm\sqrt{e^9} = \pm 90.0171$ (j) Ø (k) $e^2 - 4 = 3.3891$ (l) $\sqrt[3]{e^9} = 20.0855$ **15.** (a) 0, ln2 (b) ln5
(c) ln2, ln3 (d) 0 (e) 0, ln5 (f) ln10 **16.** (a) 4.5222 (b) 0.2643 (c) 0,0.2619 (d) –1,0.3219
(e) –1.2925,0.6610 (f) 0,1.8928 (g) 0.25,2 (h) 1 (i) 121.5 (j) 2

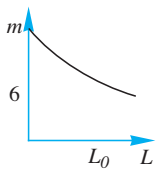
EXERCISE 7.5

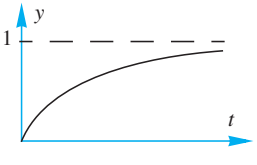
1. (a) 10 (b) 30 (c) 40 **2.** (a) 31.64 kg (b) 1.65 (c) $W = 2.4 \times 10^{0.8h}$

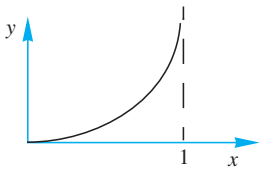
(d)


(e)


3. (a) 4.75 (b) $L = L_0 \times 10^{\left(\frac{6-m}{2.5}\right)}$ (c)


(d)


4. (a) [0,1[(b) i. 2.22 ii. 1.11 iii. 0.74 yrs (c) As c increases, reliability reduces.
(d) $x = 1 - 10^{-ct}$ (e)




5. (a) $I = \frac{a}{n^k}$ **6.** (a) 0.10 (b) $\lambda = \lambda_0 \times 10^{-kx}$ (c) 16.82% (d) $k = -\frac{1}{x}\log\left(\frac{\lambda}{\lambda_0}\right)$

EXERCISE 8.1.1

1. i. (b) 4 (c) $t_n = 4n - 2$ ii. (b) –3 (c) $t_n = -3n + 23$ iii. (b) –5 (c) $t_n = -5n + 6$ iv. (b) 0.5 (c) $t_n = 0.5n$ v. (b) 2 (c) $t_n = y + 2n - 1$ vi. (b) –2 (c) $t_n = x - 2n + 4$ **2.** –28 **3.** 9,17 **4.** –43 **5.** 7 **6.** 7 **7.** –5 **8.** 0 **9.** (a) 41 (b) 31st **10.** 2, $\sqrt{3}$ **11.** (a) i. 2 ii. –3 (b) i. 4 ii. 11 **12.** $x - 8y$ **13.** $t_n = 5 + \frac{10}{3}(n-1)$ **14.** (a) –1 (b) 0

EXERCISE 8.1.2

1. (a) 145 (b) 300 (c) –170 **2.** (a) –18 (b) 690 (c) 70.4 **3.** (a) –105 (b) 507 (c) 224 **4.** (a) 126 (b) 3900 (c) 14th week **5.** 855 4. **6.** (a) 420 (b) –210 **7.** $a = 9, b = 7$

EXERCISE 8.1.3

1. 123 **2.** –3, –0.5, 2, 4.5, 7, 9.5, 12 **3.** 3.25 **4.** $a = 3$ $d = -0.05$ **5.** 10 000 **6.** 330 **7.** –20 **8.** 328 **9.** \$725, 37wks **10.** i. \$55 ii. 2750 **11.** (a) (i) 8m (ii) 40m (b) 84m (c) Dist = $2n^2 - 2n = 2n(n-1)$ (d) 8 (e) 26 players, 1300m **12.** (a) 5050 (b) 10200 (c) 4233 **13.** (a) 145 (b) 390 (c) –1845 **14.** (b) $3n - 2$

EXERCISE 8.2.1

1. (a) $r = 2, u_5 = 48, u_n = 3 \times 2^{n-1}$ (b) $r = \frac{1}{3}, u_5 = \frac{1}{27}, u_n = 3 \times \left(\frac{1}{3}\right)^{n-1}$

(c) $r = \frac{1}{5}, u_5 = \frac{2}{625}, u_n = 2 \times \left(\frac{1}{5}\right)^{n-1}$ (d) $r = -4, u_5 = -256, u_n = -1 \times (-4)^{n-1}$

(e) $r = \frac{1}{b}, u_5 = \frac{a}{b^3}, u_n = ab \times \left(\frac{1}{b}\right)^{n-1}$ (f) $r = \frac{b}{a}, u_5 = \frac{b^4}{a^2}, u_n = a^2 \times \left(\frac{b}{a}\right)^{n-1}$ **2.** (a) ±12 (b) $\frac{\pm\sqrt{5}}{2}$ **3.** (a) ±96 (b) 15th **4.** (a) $u_n = 10 \times \left(\frac{5}{6}\right)^{n-1}$ (b) $\frac{15625}{3888} \cong 4.02$ (c) $n = 5$ (4 times)

5. $-2, \frac{4}{3}$ **6.** (a) i. \$4096 ii. \$2097.15 (b) 6.2 yrs **7.** $\left(u_n = \frac{1000}{169} \times \left(\frac{12}{5}\right)^{n-1}\right), \frac{1990656}{4225} \cong 471.16$

8. 2.5,5,10 or 10,5,2.5 **9.** 53757 **10.** 108 952 **11.** (a) \$56 156 (b) \$299 284

EXERCISE 8.2.2

1. (a) 3 (b) $\frac{1}{3}$ (c) –1 (d) $-\frac{1}{3}$ (e) 1.25 (f) $-\frac{2}{3}$ **2.** (a) 216513 (b) 1.6384×10^{-10} (c) $\frac{256}{729}$ (d) $\frac{729}{2401}$ (e) $-\frac{81}{1024}$ **3.** (a) 11; 354292 (b) 7; 473 (c) 8; 90.90909 (d) 8; 172.778 (e) 5; 2.256 (f) 13; 111.1111111111 **4.** (a) $\frac{127}{128}$ (b) $\frac{63}{8}$ (c) $\frac{130}{81}$ (d) 60 (e) $\frac{63}{64}$ **5.** 4; 118096 **6.** \$2109.50 **7.** 9.28cm **8.** (a) $V_n = V_0 \times 0.7^n$ (b) 7 **9.** 54 **10.** 53.5gms; 50 weeks. **11.** 7 **12.** 9 **13.** –0.5, –0.7797 **14.** $r = 5$, 1.8×10^{10} **15.** \$8407.35 **16.** 1.8×10^{19} or about 200 billion tonnes.

EXERCISE 8.2.3

1. Term 9 AP = 180, GP = 256. Sum to 11 terms AP = 1650, GP = 2047. **2.** 18. **3.** 12 **4.** 12, 7 **5.** 8 weeks (Ken $220 & Bo-Youn $255) **6.** (a) week 8 (b) week 12 **7.** (a) 1.618 (b) 121379 [~121400, depends on rounding errors]

EXERCISE 8.2.4

1. (i) $\frac{81}{2}$ (ii) $\frac{10}{13}$ (iii) 5000 (iv) $\frac{30}{11}$ **2.** $23\frac{23}{99}$ **3.** 6667 fish. [Nb: $t_{43} < 1$. If we use $n = 43$ then ans is 6660 fish]; 20 000 fish. Overfishing means that fewer fish are caught in the long run. [*An alternate estimate for the total catch is* 1665 *fish*.] **4.** 27 **5.** 48,12,3 or 16,12,9 **6.** (a) $\frac{11}{30}$ (b) $\frac{37}{99}$ (c) $\frac{191}{90}$ **7.** 128 cm **8.** $\frac{121}{9}$ **9.** $2+\frac{4}{3}\sqrt{3}$ **10.** $\frac{1-(-t)^n}{1+t}$ $\frac{1}{1+t}$ **11.** $\frac{1-(-t^2)^n}{1+t^2}$ $\frac{1}{1+t^2}$

EXERCISE 8.2.5

1. 3, –0.2 **2.** $\frac{2560}{93}$ **3.** $\frac{10}{3}$ **4.** (a) $\frac{43}{18}$ (b) $\frac{458}{99}$ (c) $\frac{413}{990}$ **5.** 9900 **6.** 3275 **7.** 3

8. $t_n = 6n - 14$ **9.** 6 **10.** $-\frac{1}{6}$ **11.** i. 12 ii. 26 **12.** 9, 12 **13.** ±2 **14.** (5, 5, 5), (5, –10, 20)

15. (a) 2, 7 (b) 2, 5, 8 (c) $3n - 1$ **16.** (a) 5 (b) 2 m

EXERCISE 8.3

1. $2773.08 **2.** $4377.63 **3.** $1781.94 **4.** $12216 **5.** $35816.95 **6.** $40349.37 **7.** $64006.80 **8.** $276971.93, $281325.41 **9.** $63762.25 **10.** $98.62, $9467.14, interest $4467.14. Flat interest = $6000 **11.** $134.41, $3790.44, 0.602% /month (or 7.22% p.a)

EXERCISE 9.1

1.	***a* cm**	***b* cm**	***c* cm**	***A***	***B***	***C***
1	3.8	4.1	1.6	67°	90°	23°
2	81.5	98.3	55.0	56°	90°	34°
3	32.7	47.1	33.9	44°	90°	46°
4	1.61	30.7	30.7	3°	90°	87°
5	2.3	2.74	1.49	57°	90°	33°
6	48.5	77	59.8	39°	90°	51°
7	44.4	81.6	68.4	33°	90°	57°
8	2.93	13.0	12.7	13°	90°	77°
9	74.4	94.4	58.1	52°	90°	38°
10	71.8	96.5	64.6	48°	90°	42°
11	23.3	34.1	24.9	43°	90°	47°
12	43.1	43.2	2.3	87°	90°	3°
13	71.5	80.2	36.4	63°	90°	27°
14	33.5	34.1	6.5	79°	90°	11°
15	6.1	7.2	3.82	58°	90°	32°
16	29.1	30	7.3	76°	90°	14°
17	29.0	29.1	2.0	86°	90°	4°
18	34.5	88.2	81.2	23°	90°	67°
19	24.0	29.7	17.5	54°	90°	36°
20	41.2	46.2	21.0	63°	90°	27°
21	59.6	72.9	41.8	55°	90°	35°
22	5.43	6.8	4.09	53°	90°	37°

23	13.0	19.8	14.9	41°	90°	49°
24	14.0	21.3	16.1	41°	90°	49°
25	82.4	88.9	33.3	68°	90°	22°

2. (a) $2\sqrt{3}$ (b) $5(1+\sqrt{3})$ (c) 4 (d) $2(1+\sqrt{3})$ (e) $\frac{4}{3}(3+\sqrt{3})$ (f) $\sqrt{106}-5$ **4.** (a) $25(1+\sqrt{3})$ (b) $\frac{40\sqrt{3}}{3}$

EXERCISE 9.2

1. (a) i. 030°T ii. 330°T iii. 195°T iv. 200°T (b) i. N25°E ii. S iii. S40°W iv. N10°W **2.** 37.49m **3.** 18.94m **4.** 37° 18' **5.** $\frac{26}{9}$ m/s **6.** N58° 33'W, 37.23 km **7.** 199.82 m **8.** 10.58 m **9.** 72.25 m **10.** 25.39 km **11.** 15.76 m **12.** (a) 3.01km N, 3.99km E (b) 2.87km E 0.88km S (c) 6.86km E 2.13km N (d) 7.19km 253°T **13.** 524m

EXERCISE 9.3

1. (a) 39°48' (b) 64°46' **2.** (a) 12.81 cm (b) 61.35 cm (c) 77°57' (d) 60.83 cm (e) 80° 32' **3.** (a) 21°48' (b) 42°2' (c) 26°34' **4.** (a) 2274 (b) 12.7° **5.** 251.29 m **6.** (a) 103.5 m (b) 35.26° (c) 39.23° **7.** (b) 53.43 (c) 155.16 m (d) 145.68 m **8.** (b) 48.54 m **9.** (a) $\sqrt{(b-c)^2+h^2}$ (b) $\tan^{-1}\left(\frac{h}{a}\right)$ (c) $\tan^{-1}\left(\frac{h}{b-c}\right)$ (d) $2(b+c)\sqrt{h^2+a^2}+2a\sqrt{(b-c)^2+h^2}$ **10.** 82.80 m **11.** (a) 40.61 m (b) 49.46 m **12.** (a) 10.61 cm (b) 75° 58' (c) 93° 22' **13.** (a) 1.44 m (b) 73° 13' (c) 62° 11'

EXERCISE 9.4

1. (a) 1999.2 cm^2 (b) 756.8 cm^2 (c) 3854.8 cm^2 (d) 2704.9 cm^2 (e) 538.0 cm^2 (f) 417.5 cm^2 (g) 549.4 cm^2 (h) 14.2 cm^2 (i) 516.2 cm^2 (j) 281.5 cm^2 (k) 918.8 cm^2 (l) 387.2 cm^2 (m) 139.0 cm^2 (n) 853.7 cm^2 (o) 314.6 cm^2 **2.** 69345 m^2 **3.** $100\pi - 6\sqrt{91}$ cm^2 **4.** 17.34 cm **5.** (a) 36.77sq units (b) 14.70 sq units (c) 62.53 sq units **6.** 52.16 cm^2 **7.** 27° 2' **8.** $\frac{(b+a\times\tan\theta)^2}{2\tan\theta}$ **9.** Area of Δ ACD = 101.78 cm^2, area of Δ ABC = 61.38 cm^2

EXERCISE 9.5.1

	a cm	*b* cm	*c* cm	*A*	*B*	*C*
1	13.3	37.1	48.2	10°	29°	141°
2	2.7	1.2	2.8	74°	25°	81°
3	11.0	0.7	11.3	60°	3°	117°
4	31.9	39.1	51.7	38°	49°	93°
5	18.5	11.4	19.5	68°	35°	77°
6	14.6	15.0	5.3	75°	84°	21°
7	26.0	7.3	26.4	79°	16°	85°
8	21.6	10.1	28.5	39°	17°	124°
9	0.8	0.2	0.8	82°	16°	82°
10	27.7	7.4	33.3	36°	9°	135°
11	16.4	20.7	14.5	52°	84°	44°
12	21.4	45.6	64.3	11°	24°	145°
13	30.9	27.7	22.6	75°	60°	45°

14	29.3	45.6	59.1	29°	49°	102°
15	9.7	9.8	7.9	65°	67°	48°
16	21.5	36.6	54.2	16°	28°	136°
17	14.8	29.3	27.2	30°	83°	67°
18	10.5	0.7	10.9	52°	3°	125°
19	11.2	6.9	17.0	25°	15°	140°
20	25.8	18.5	40.1	30°	21°	129°

EXERCISE 9.5.2

	a	*b*	*c*	*A*°	*B*°	*C*°	*c**	*B**°	*C**°
1	7.40	18.10	21.06	20.00	56.78	103.22	12.95	123.22	36.78
2	13.30	19.50	31.36	14.00	20.77	145.23	6.49	159.23	6.77
3	13.50	17.00	25.90	28.00	36.24	115.76	4.12	143.76	8.24
4	10.20	17.00	25.62	15.00	25.55	139.45	7.22	154.45	10.55
5	7.40	15.20	19.55	20.00	44.63	115.37	9.02	135.37	24.63
6	10.70	14.10	21.41	26.00	35.29	118.71	3.94	144.71	9.29
7	11.50	12.60	22.94	17.00	18.68	144.32	1.16	161.32	1.68
8	8.30	13.70	18.67	24.00	42.17	113.83	6.36	137.83	18.17
9	13.70	17.80	30.28	14.00	18.32	147.68	4.27	161.68	4.32
10	13.40	17.80	26.19	28.00	38.58	113.42	5.24	141.42	10.58
11	12.10	16.80	25.63	23.00	32.85	124.15	5.30	147.15	9.85
12	12.00	14.50	24.35	21.00	25.66	133.34	2.72	154.34	4.66
13	12.10	19.20	29.34	16.00	25.94	138.06	7.57	154.06	9.94
14	7.20	13.10	19.01	15.00	28.09	136.91	6.30	151.91	13.09
15	12.20	17.70	23.73	30.00	46.50	103.50	6.93	133.50	16.50
16	9.20	20.90	27.97	14.00	33.34	132.66	12.59	146.66	19.34
17	10.50	13.30	21.96	20.00	25.67	134.33	3.03	154.33	5.67
18	9.20	19.20	26.29	15.00	32.69	132.31	10.80	147.31	17.69
19	7.20	13.30	18.33	19.00	36.97	124.03	6.82	143.03	17.97
20	13.50	20.40	25.96	31.00	51.10	97.90	9.01	128.90	20.10
21	10.80	20.80	24.48	26.00	57.59	96.41	12.91	122.41	31.59
22	13.00	12.20	23.91	19.00	17.79	143.21	0.84	162.21	1.21
23	13.60	20.40	22.92	36.00	61.85	82.15	10.09	118.15	25.85
24	11.40	12.50	22.88	16.00	17.59	146.41	1.15	162.41	1.59
25	8.00	16.80	23.99	10.00	21.39	148.61	9.10	158.61	11.39

26. (a-d) no triangles exist.

EXERCISE 9.5.3

1. 30.64 km **2.** 4.57 m **3.** 476.4 m **4.** 201°47'T **5.** 222.9 m **6.** (a) 3.40 m (b) 3.11 m
7. (b) 1.000 m (c) 1.715 m **8.** (a) 51.19 min (b)1 hr 15.96 min (c) 14.08 km **9.** $4886 **10.**906 m

EXERCISE 9.5.4

	a cm	*b* cm	*c* cm	*A*	*B*	*C*
1	13.5	9.8	16.7	54°	36°	90°
2	8.9	10.8	15.2	35°	44°	101°
3	22.8	25.6	12.8	63°	87°	30°
4	21.1	4.4	21.0	85°	12°	83°
5	15.9	10.6	15.1	74°	40°	66°
6	8.8	13.6	20.3	20°	32°	128°
7	9.2	9.5	13.2	44°	46°	90°
8	23.4	62.5	58.4	22°	89°	69°

9	10.5	9.6	15.7	41°	37°	102°
10	21.7	36.0	36.2	35°	72°	73°
11	7.6	3.4	9.4	49°	20°	111°
12	7.2	15.2	14.3	28°	83°	69°
13	9.1	12.5	15.8	35°	52°	93°
14	14.9	11.2	16.2	63°	42°	75°
15	2.0	0.7	2.5	38°	13°	129°
16	7.6	3.7	9.0	56°	24°	100°
17	18.5	9.8	24.1	45°	22°	113°
18	20.7	16.3	13.6	87°	52°	41°
19	14.6	22.4	29.9	28°	46°	106°
20	7.0	6.6	9.9	45°	42°	93°
21	21.8	20.8	23.8	58°	54°	68°
22	1.1	1.7	1.3	41°	89°	50°
23	1.2	1.2	0.4	85°	76°	19°
24	23.7	27.2	29.7	49°	60°	71°
25	3.4	4.6	5.2	40°	60°	80°

EXERCISE 9.5.5

1. (a) 10.14 km (b) 121°T **2.** 7° 33' **3.** 4.12 cm **4.** 57.32 m **5.** 315.5 m **6.** (a) 124.3 km (b) W28° 47' S

EXERCISE 9.5.6

1. 39.60m 52.84m **2.** 30.2m **3.** 54°,42°, 84° **4.** 37° **5.** 028°T. **6.** 108.1cm **7.** (i) 135° (ii) 136cm **8.** 41°, 56°, 83° **9.** (i) 158° left (ii) 43.22km **10.** 264m **11.** 53.33cm **12.** 186m **13.** 50.12cm **14.** 5.17cm **15.** (a) 5950m (b) 13341m (c) 160° (d) 243° **17.** (a) 20.70° (b) 2.578 m (c) 1.994 m^3 **18.** (a) 4243 m^2 (b) 86 m (c) 101 m

EXERCISE 9.6

1. 5.36 cm **2.** 12.3 m **3.** 24 m **4.** 40.3 m, 48.2° **5.** 16.5 min, 8.9° **6.** ~10:49 am

7. (a) i. $\frac{d\sin\phi}{\sin(\phi-\theta)}$ ii. $\frac{d\sin\theta}{\sin(\phi-\theta)}$ (b) $\frac{d\sin\phi\tan\alpha}{\sin(\phi-\theta)}$ or $\frac{d\sin\theta\tan\beta}{\sin(\phi-\theta)}$ (c) $d\left(\frac{\sin\phi\cos\theta}{\sin(\phi-\theta)}-1\right)$

EXERCISE 9.7

1. (i) $\frac{169\pi}{150}\text{cm}^2$, $5.2+\frac{13\pi}{15}\text{cm}$ (ii) $\frac{529\pi}{32}\text{cm}^2$, $23+\frac{23\pi}{8}\text{cm}$ (iii) $242\pi\text{cm}^2$, $88+11\pi\text{cm}$

(iv) $\frac{1156\pi}{75}\text{m}^2$, $13.6+\frac{68\pi}{15}\text{m}$ (v) $\frac{96\pi}{625}\text{cm}^2$, $1.28+\frac{12\pi}{25}\text{cm}$ (vi) $\frac{361\pi}{15}\text{cm}^2$, $15.2+\frac{19\pi}{3}\text{cm}$

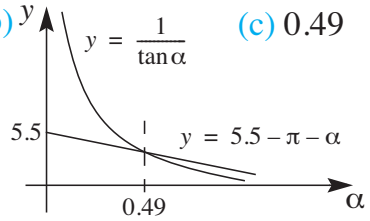
(vii) $5248.8\pi\text{m}^2$, $648+32.4\pi\text{cm}$ (viii) $\frac{12943\pi}{300}\text{cm}^2$, $17.2+\frac{301\pi}{30}\text{cm}$

(ix) $\frac{1922\pi}{75}\text{cm}^2$, $12.4+\frac{124\pi}{15}\text{cm}$ (x) $\frac{15884\pi}{3}\text{cm}^2$, $152+\frac{418\pi}{3}\text{cm}$ (xi) $12\pi\text{cm}^2$, $24+2\pi\text{cm}$

(xii) $\frac{98\pi}{3}\text{cm}^2$, $28+\frac{14\pi}{3}\text{cm}$ (xiii) $\frac{196\pi}{75}\text{cm}^2$, $5.6+\frac{28\pi}{15}\text{cm}$ (xiv) $\frac{11532\pi}{25}\text{cm}^2$, $49.6+\frac{186\pi}{5}\text{cm}$

(xv) $\frac{3\pi}{50}\text{cm}^2$, $2.4+\frac{\pi}{10}\text{cm}$ **2.** 0.63^c, 36° **3.** 0.0942m^3 **4.** 1.64^c **5.** 79cm. **6.** 5.25cm^2

7. (a) 31.83m (b) 406.28m (c) 11° **8.** 1.11^c **9.** 0.75^c **10.** (a) 1.85^c (b) i. 37.09 cm ii. 88.57 cm (c) 370.92cm^2 **11.** 26.57 cm^2 **12.** 193.5 cm **13.** (a) 105.22 cm (b) 118.83 cm **14.** (a) 9 cm

(b) 12 cm (c) 36°52′ **15.** (b) 

(c) 0.49 **16.** 1439.16 cm²

EXERCISE 10.1

1. (a) 120° (b) 108° (c) 216° (d) 50° **2.** (a) π^c (b) $\frac{3\pi^c}{2}$ (c) $\frac{7\pi^c}{9}$ (d) $\frac{16\pi^c}{9}$ **3.** (a) $\frac{\sqrt{3}}{2}$ (b) $-\frac{1}{2}$

(c) $-\sqrt{3}$ (d) $-\frac{1}{2}$ (e) $-\frac{\sqrt{3}}{2}$ (f) $\frac{1}{\sqrt{3}}$ (g) $-\frac{1}{\sqrt{2}}$ (h) $-\frac{1}{\sqrt{2}}$ (i) 1 (j) $-\frac{1}{\sqrt{2}}$ (k) $\frac{1}{\sqrt{2}}$ (l) –1 (m) 0 (n) 1 (o) 0

4. (a) 0 (b) –1 (c) 0 (d) $\frac{1}{\sqrt{2}}$ (e) $-\frac{1}{\sqrt{2}}$ (f) –1 (g) $-\frac{1}{2}$ (h) $-\frac{\sqrt{3}}{2}$ (i) $\frac{1}{\sqrt{3}}$ (j) $-\frac{\sqrt{3}}{2}$ (k) $\frac{1}{2}$ (l) $-\sqrt{3}$

(m) $-\frac{1}{\sqrt{2}}$ (n) $\frac{1}{\sqrt{2}}$ (o) –1 **5.** (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) $\frac{1}{2}$ **6.** (a) $-\frac{1}{2}$ (b) $-\frac{1}{\sqrt{2}}$ (c) $\sqrt{3}$ (d) $\frac{1}{2}$ (e) $-\frac{\sqrt{3}}{2}$

(f) $\frac{1}{\sqrt{3}}$ (g) $-\frac{\sqrt{3}}{2}$ **7.** (a) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ (b) $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ (c) $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ (d) $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ **8.** (a) 0 (b) $\frac{\sqrt{3}}{2}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1+\sqrt{3}}{2\sqrt{2}}$ **10.** (a) $-\frac{2}{3}$ (b) $-\frac{2}{3}$ (c) $-\frac{2}{3}$ **11.** (a) $-\frac{2}{5}$ (b) $\frac{2}{5}$ **12.** (a) k (b) $-\frac{1}{k}$ (c) $-k$

13. (a) $\frac{\sqrt{5}}{3}$ (b) $-\frac{\sqrt{5}}{3}$ **14.** (a) $-\frac{3}{5}$ (b) $\frac{3}{4}$ (c) $\frac{4}{5}$ **15.** (a) $\frac{4}{5}$ (b) $\frac{3}{4}$ **16.** (a) $-k$ (b) $-\sqrt{1-k^2}$

17. (a) $-\sqrt{1-k^2}$ (b) $k/\sqrt{1-k^2}$ **18.** (a) $\sin\theta$ (b) $\cot\theta$ (c) 1

19. (a) $\frac{\pi}{3}, \frac{2\pi}{3}$ (b) $\frac{\pi}{3}, \frac{5\pi}{3}$ (c) $\frac{\pi}{3}, \frac{4\pi}{3}$ (d) $\frac{5\pi}{6}, \frac{7\pi}{6}$ (e) $\frac{5\pi}{6}, \frac{11\pi}{6}$ (f) $\frac{7\pi}{6}, \frac{11\pi}{6}$

EXERCISE 10.2.1

3. (a) $x^2 + y^2 = k^2, -k \le x \le k$ (b) $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1, -b \le x \le b$ (c) $(x-1)^2 + (2-y)^2 = 1, 0 \le x \le 2$

(d) $\frac{(1-x)^2}{b^2} + \frac{(y-2)^2}{a^2} = 1$ (e) $5x^2 + 5y^2 + 6xy = 16$ **4.** (a) (i) $-\frac{4}{5}$ (ii) $-\frac{5}{3}$ (b) (i) $\frac{4}{\sqrt{7}}$ (ii) $-\frac{\sqrt{7}}{3}$

5. (a) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ (b) $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$ (c) $0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, 2\pi$ (d) $\frac{\pi}{2}, \frac{3\pi}{2}$ **9.** (a) $\frac{2a}{a^2+1}$ (b) $\frac{a^2-1}{a^2+1}$

10. (a) i. 1 ii. 1 (b) 1 **11.** (a) $\frac{1-\sqrt{x^2-1}}{x}$ (b) $\frac{1+\sqrt{x^2-1}}{x}$ (c) $\frac{2}{x^2} - 1$ **12.** (a) $5 \le \cos^2\theta + 5 \le 6$

(b) $1 \le \frac{5}{3\sin^2\theta + 2} \le \frac{5}{2}$ (c) $-2 \le 2\cos^2\theta + \sin\theta - 1 \le \frac{9}{8}$ **13.** (a) ±2 (b) $\frac{\pi}{6} + 2k\pi, k \in \mathbb{Z}$ or

$\frac{7\pi}{6} + 2k\pi, k \in \mathbb{Z}$ **14.** (a) $5^{-4} \le 5^{3\sin\theta - 1} \le 25$ (b) $3^{-1} \le 3^{1-2\cos\theta} \le 27$ **15.** (a) $1 + 2k$

(b) $(1-k)\sqrt{1+2k}$ **16.** (a) $\frac{1-a}{2\sqrt{a}}$ (b) i. $2 + \sqrt{2a-a^2}$ ii. $\frac{-\sqrt{2a-a^2}}{1-a}$ **17.** (a) $\frac{2}{3}$ (b) $0, \pm\frac{2\sqrt{2}}{3}$

18. $0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi$

EXERCISE 10.2.2

1. (a) $-\frac{5\sqrt{11}}{18}$ (b) $-\frac{7}{18}$ (c) $\frac{5\sqrt{11}}{7}$ (d) $\frac{35\sqrt{11}}{162}$ **2.** (a) $-\frac{3}{5}$ (b) $-\frac{4}{5}$ (c) $\frac{3}{4}$ (d) $\frac{24}{7}$ **3.** $\frac{(\sqrt{3}-1)\sqrt{2}}{4}$

4. (a) $\frac{2ab}{a^2+b^2}$ (b) $\frac{a^2+b^2}{2ab}$ (c) $\frac{a^4-6a^2b^2+b^4}{(a^2+b^2)^2}$ (d) $\frac{2ab}{b^2-a^2}$ **7.** (a) $0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$ (b) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

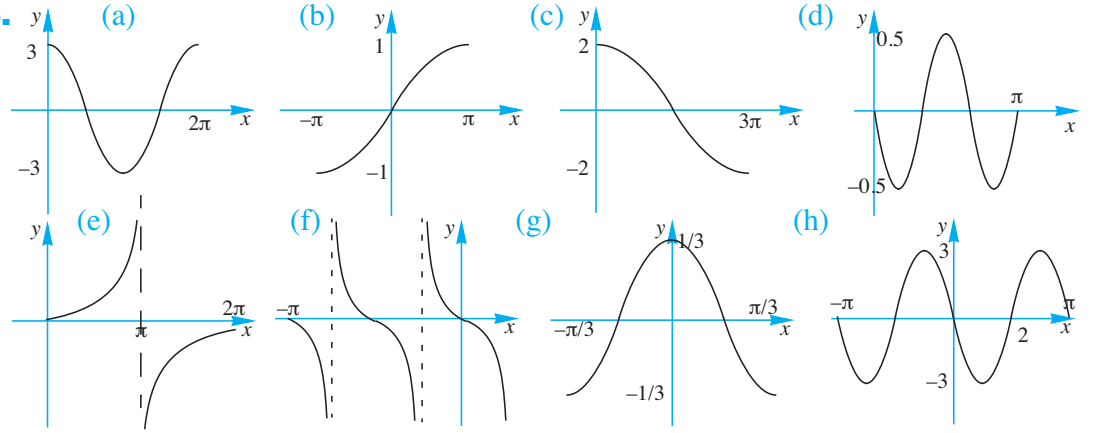
(c) $0, \pi, 2\pi, \alpha, \pi \pm \alpha, 2\pi - \alpha, \alpha = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

EXERCISE 10.3

1. (a) 4π (b) $\frac{2\pi}{3}$ (c) 3π (d) 4π (e) 2 (f) $\frac{\pi}{2}$ **2.** (a) 5 (b) 3 (c) 5 (d) 0.5 **3.** (a) 2π, 2 (b) 6π, 3 (c) π

(d) π (e) π, 4 (f) π, 3 (g) 6π (h) $\frac{2\pi}{3}, \frac{1}{4}$ (i) 3π (j) $\frac{8\pi}{3}, \frac{2}{3}$

4.



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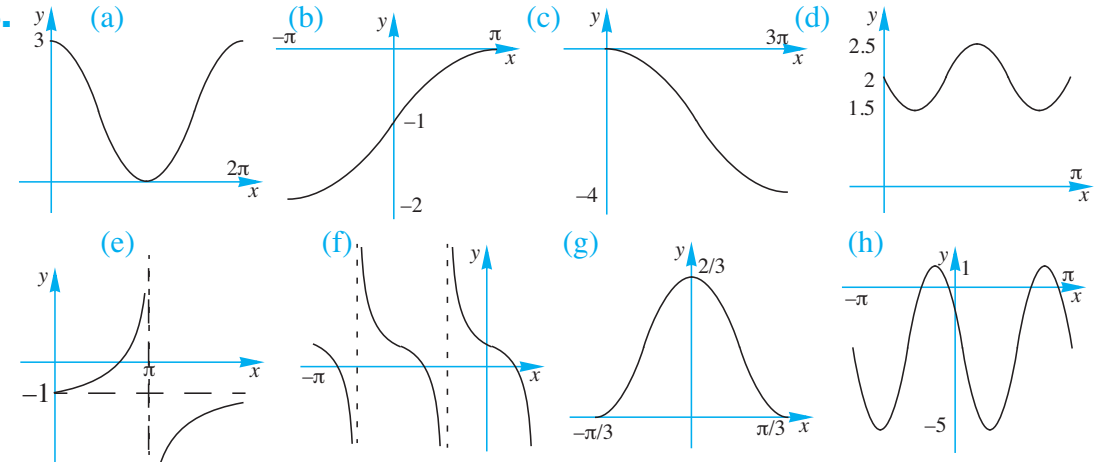

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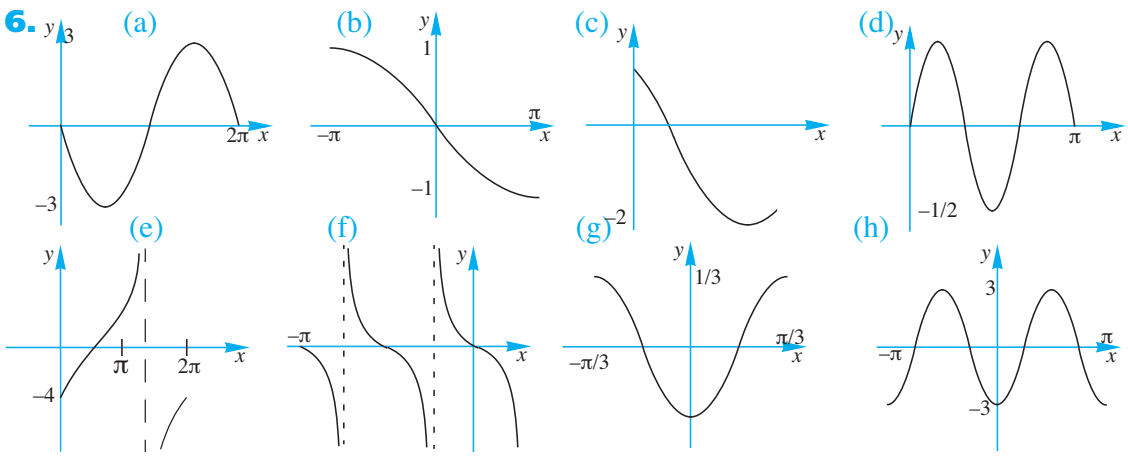
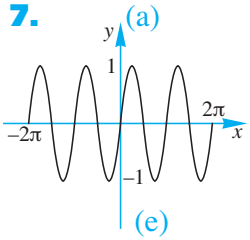
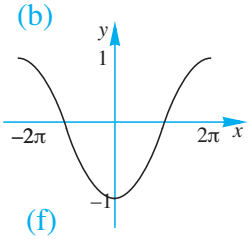
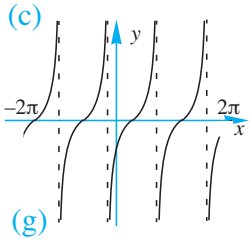
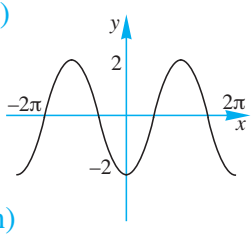
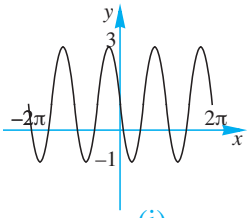


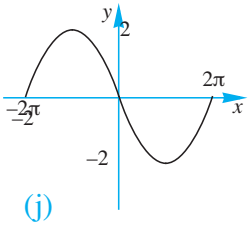
7. (a) 

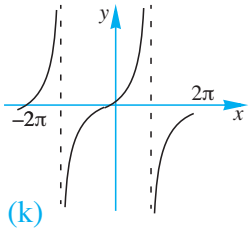
(b) 

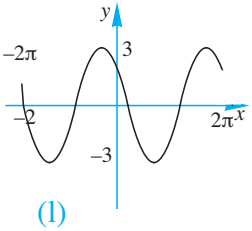
(c) 

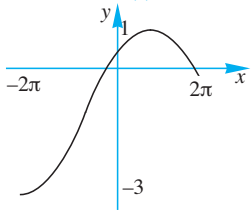
(d) 

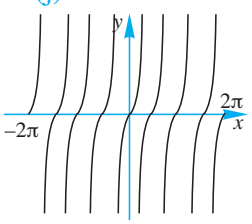
(e) 

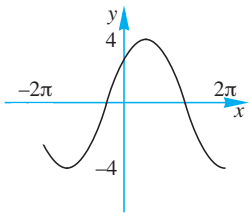
(f) 

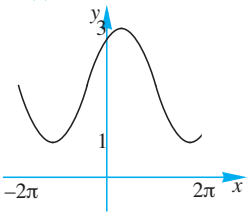
(g) 

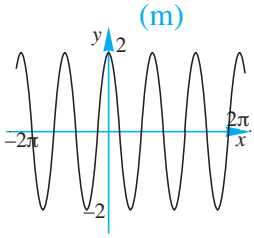
(h) 

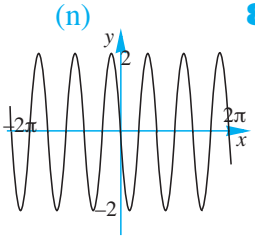
(i) 

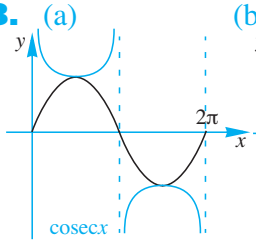
(j) 

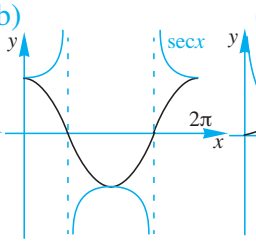
(k) 

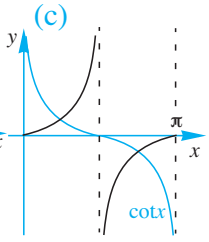
(l) 

(m) 

(n) 

8. (a) 

(b) 

(c) 

EXERCISE 10.4

1. (a) $\frac{\pi}{4}, \frac{3\pi}{4}$ (b) $\frac{7\pi}{6}, \frac{11\pi}{6}$ (c) $\frac{\pi}{3}, \frac{2\pi}{3}$ (d) $\frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}$ (e) $\frac{\pi}{3}, \frac{5\pi}{3}$ (f) $\frac{5}{4}, \frac{7}{4}, \frac{13}{4}, \frac{15}{4}$, $\frac{21}{4}, \frac{23}{4}$ **2.** (a) $\frac{\pi}{4}, \frac{7\pi}{4}$ (b) $\frac{2\pi}{3}, \frac{4\pi}{3}$ (c) $\frac{\pi}{6}, \frac{11\pi}{6}$ (d) π (e) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ (f) $\frac{3}{2}, \frac{5}{2}, \frac{11}{2}$ **3.** (a) $\frac{\pi}{6}, \frac{7\pi}{6}$ (b) $\frac{3\pi}{4}, \frac{7\pi}{4}$ (c) $\frac{\pi}{3}, \frac{4\pi}{3}$ (d) $4\tan^{-1}2$ (e) $\frac{\pi}{3}, \frac{5\pi}{6}, \frac{4\pi}{3}, \frac{11\pi}{6}$ (f) 3 **4.** (a) 90°, 330° (b) 180°,240° (c) 90°,270° (d) 65°,155° (e) $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$ (f) 0, π, 2π **5.** (a) 60°,300° (b) $\frac{4\pi}{3}, \frac{5\pi}{3}$ (c) $\frac{\pi}{6}, \frac{7\pi}{6}$ (d) 23°35',156°25' (e) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ (f) $\frac{2\pi}{3}, \frac{5\pi}{3}$ (g) $\frac{5\pi}{6}, \frac{9\pi}{6}$ (h) $3.3559^c, 5.2105^c$ (i) $\frac{\pi}{3}, \frac{4\pi}{3}$ (j) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ (k) $\frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}$ (l) 68°12',248°12' **6.** (a) $-\frac{3\pi}{4}, \frac{\pi}{4}$ (b) $\pm\frac{\pi}{3}$ (c) $-\frac{7\pi}{8}, -\frac{3\pi}{8}, \frac{\pi}{8}, \frac{5\pi}{8}$ (d) $-\frac{\pi}{2}$ (e) $\pm\frac{\pi}{2}$ (f) $\frac{\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{15\pi}{8}$ (g) $\frac{\pi}{2}, \frac{3\pi}{2}$ (h) $\frac{\pi}{2}, \frac{3\pi}{2}$

7. (a) $\frac{3\pi}{4}, \frac{7\pi}{4}, \tan^{-1}\left(\frac{2}{3}\right), \pi + \tan^{-1}\left(\frac{2}{3}\right)$ (b) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{4}$

8. (a) $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$ (b) $\frac{2\pi}{3}, \frac{4\pi}{3}$ (c) 0,1,2,3,4,5,6

9. i. $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right) \cup \left(\frac{13\pi}{6}, \frac{17\pi}{6}\right)$ ii. $\left(\pi + \sin^{-1}\left(\frac{1}{\sqrt{3}}\right), 2\pi - \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)\right) \cup \left(3\pi + \sin^{-1}\left(\frac{1}{\sqrt{3}}\right), 4\pi - \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)\right)$

10. (a) ii. $\left[0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right]$ (b) ii. $\left[0, \frac{\pi}{6}\right) \cup \left(\frac{\pi}{2}, \frac{5\pi}{6}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right]$ **11.** (a) 90°,199°28',340°32' (b)]199°28',340°32'[

EXERCISE 10.5

1. (a) 5, 24, 11, 19 (b) $T = 5\sin\left(\frac{\pi t}{12} - 3\right) + 19$ (c) 23.6° **2.** (a) 3. 4.2, 2, 7

(b) $L = 3\sin\left(\frac{\pi t}{2.1} - 3\right) + 7$ **3.** (a) 5, 11, 0, 7 (b) $V = 5\sin\left(\frac{2\pi t}{11}\right) + 7$ **4.** (a) 1, 11, 1, 12

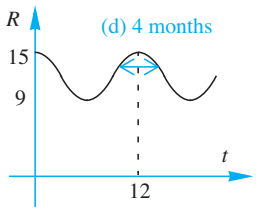
(b) $P = \sin\left(\frac{2\pi}{11}(t-1)\right) + 12$ **5.** (a) 2.6, 7, 2, 6 (b) $S = 2.6\sin\frac{2\pi}{7}(t-2) + 6$ **6.** (a) 0.6, 3.5, 0, 11

(b) $P = 0.6\sin\left(\frac{4\pi t}{7}\right) + 11$ **7.** (a) 0.8, 4.6, 2.7, 11 (b) $D = 0.8\sin\frac{\pi}{2.3}(t - 2.7) + 11$ **8.** (a) 3000

(b) 1000, 5000 (c) $\frac{4}{9}$ **9.** (a) 6.5 m, 7.5 m (b) 1.58 sec, 3.42 sec **10.** (a) 750, 1850 (b) 3.44

(c) Mid-April to End of August

11. (a) 15000 (b) 12 months (c) (d) 4 months

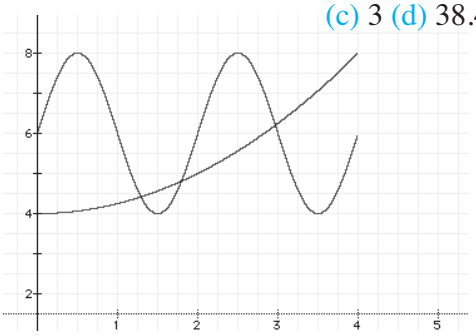


12. (a) π,–2,2 (b) $\frac{1}{3}$ m (c) $\frac{4}{3}$ m

13.(a)

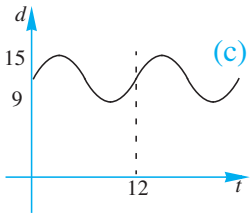
t	0	0.5	1	1.5	2	2.5	3	3.5	4
$F(t)$	6	8	6	4	6	8	6	4	6
$G(t)$	4	4.0625	4.25	4.5625	5	5.5625	6.25	7.0625	8

(b)



(c) 3 (d) 38.45%

14. (a)



(b) i. 7,11,19,23

ii. $[0, 7] \cup [11, 19] \cup [23, 24]$

(c) 14.9m

EXERCISE 11.1.1

1. (i) $\begin{bmatrix} 2 & 8 \\ -2 & 16 \end{bmatrix}$ (ii) $\begin{bmatrix} -2 & 3 \\ 1 & -5 \end{bmatrix}$ (iii) $\begin{bmatrix} 6 & -9 \\ -3 & 15 \end{bmatrix}$ (iv) $\begin{bmatrix} 4 & 5 \\ -3 & 21 \end{bmatrix}$ (v) $\begin{bmatrix} 7 & 6 \\ -5 & 34 \end{bmatrix}$ (vi) $\begin{bmatrix} -2 & -19 \\ 3 & -27 \end{bmatrix}$

2. (i) $\begin{bmatrix} -1 & -1 & -2 \\ 0 & 6 & -9 \end{bmatrix}$ (ii) $\begin{bmatrix} -2 & -2 & -4 \\ 0 & 12 & -18 \end{bmatrix}$ (iii) $\begin{bmatrix} -2 & -5 \\ 2 & 2 \\ 3 & 1 \end{bmatrix}$ (iv) $\begin{bmatrix} 2 & 0 \\ 4 & 2 \\ 0 & -2 \end{bmatrix}$ (v) $\begin{bmatrix} 5 & 5 \\ 4 & 1 \\ -3 & -4 \end{bmatrix}$ (vi) $\begin{bmatrix} -6 & -15 \\ 6 & 6 \\ 9 & 3 \end{bmatrix}$

3. (i) $\begin{bmatrix} 3 & 3 & 6 \\ 0 & 3 & 12 \\ 0 & 6 & 3 \end{bmatrix}$ (ii) $\begin{bmatrix} -2 & -4 & 6 \\ 4 & 0 & 2 \\ 4 & 0 & -8 \end{bmatrix}$ (iii) $\begin{bmatrix} 0 & -1 & 5 \\ 2 & 1 & 5 \\ 2 & 2 & -3 \end{bmatrix}$ (iv) $\begin{bmatrix} 2 & 3 & -1 \\ -2 & 1 & 3 \\ -2 & 2 & 5 \end{bmatrix}$ (v) $\begin{bmatrix} 1 & -1 & 12 \\ 4 & 3 & 14 \\ 4 & 6 & -5 \end{bmatrix}$ (vi) $\begin{bmatrix} -3 & -5 & 4 \\ 4 & -1 & -2 \\ 4 & -2 & -9 \end{bmatrix}$

4. (i) 23 (ii) Nuts (iii) Taps (iv) Week 2 Wednesday - = B3 + I3, = B4 +I4 **5.** 7 by 5 **6.** $\begin{bmatrix} 3a & 3 \\ 0 & 0 \end{bmatrix}$

7. 3 **8.** 2 **9.** (a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} -\cos 2\theta & 0 \\ 0 & \cos 2\theta \end{bmatrix}$ (c) $\begin{bmatrix} -\cos 2\theta & 0 \\ 0 & \cos 2\theta \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **10.** (a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} -\cos 2\theta & 2\sin\theta \\ -2\cos\theta & \cos 2\theta \end{bmatrix}$ **11.** $a = 3, b = 4$ **12.** $x = 1$

EXERCISE 11.1.2

1. (i) $\begin{bmatrix} 5 & -4 \\ 2 & -1 \end{bmatrix}$ (ii) $\begin{bmatrix} 6 & -8 \\ -15 & 26 \end{bmatrix}$ (iii) $\begin{bmatrix} -7 & 17 \\ 3 & 3 \end{bmatrix}$ (iv) $\begin{bmatrix} -15 & -10 \\ -10 & -36 \end{bmatrix}$ (v) $\begin{bmatrix} \frac{19}{6} & \frac{89}{3} \\ -\frac{1}{8} & \frac{11}{2} \end{bmatrix}$ (vi) $\begin{bmatrix} 2.6 & 5.5 \\ -8.55 & 0 \end{bmatrix}$

(vii) $\begin{bmatrix} 12 & -4 & 2 \\ 6 & 0 & -3 \end{bmatrix}$ (viii) $\begin{bmatrix} -9 & 3 & 2 \\ 4 & -2 & -2 \end{bmatrix}$ (ix) $\begin{bmatrix} \frac{7}{6} & \frac{3}{2} & \frac{7}{3} \\ \frac{13}{3} & 5 & \frac{29}{3} \end{bmatrix}$ (x) $\begin{bmatrix} 1.6 & -0.7 & 3.8 \\ -3.9 & -4.2 & -5.7 \end{bmatrix}$ (xi) $\begin{bmatrix} 11 & -2 \\ 11 & 3 \\ 1 & -7 \end{bmatrix}$

(xii) $\begin{bmatrix} 7 & 5 \\ 21 & -26 \\ 6 & 4 \end{bmatrix}$ (xiii) $\begin{bmatrix} 12 & 6 & 2 \\ 12 & 1 & 4 \\ 21 & 3 & 6 \end{bmatrix}$ (xiv) $\begin{bmatrix} -5 & 6 & 1 \\ 4 & -4 & -12 \\ -1 & 4 & 2 \end{bmatrix}$ (xv) $\begin{bmatrix} -7 & 2 & 13 \\ 8 & -8 & -17 \\ -4 & -11 & 4 \end{bmatrix}$ (xvi) $\begin{bmatrix} -\frac{25}{2} & -\frac{50}{3} & 4 \\ -\frac{79}{4} & -\frac{83}{6} & \frac{27}{4} \\ -2 & \frac{11}{6} & \frac{3}{4} \end{bmatrix}$

(xvii) $\begin{bmatrix} x + 3x^2 & -x^2 + 1 \\ 2x + 5x^2 & -x^2 + 2 \\ -x^2 + 5x & -2x \end{bmatrix}$ (xviii) $\begin{bmatrix} a + 2x^2 - a^2 & 2a + 4x & -2x^2 + 2a \\ a - 2ax & -2a & 3a + 2ax \\ 0 & 2a & -ax - x \end{bmatrix}$

2. (i) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ (iii) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ if n is odd and $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ if n is even.

3. (i) $\begin{bmatrix} 0 & 3 & 4 & 2 \\ 1 & 0 & 0 & 6 \\ 1 & 2 & 0 & 2 \\ 4 & 3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 120 \\ 105 \\ 110 \\ 100 \end{bmatrix}$ (ii) $\begin{bmatrix} 955 \\ 720 \\ 530 \\ 795 \end{bmatrix}$ (iii) Loading costs for each depot. **4.** $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ $n = 4$.

5. (a) 2×1 (b) – (c) 3×2 (d) – (e) – (f) 1×3 **6.** (a) No (b) Yes (c) $AB = BA$ **7.** $\begin{bmatrix} n \\ \dfrac{12 - 5n}{4} \end{bmatrix}, n \in \mathbb{R}$

8. $a = -0.4, b = 2$ **9.** B **10.** (a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, S^3 = S$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, S^{2n+1} = S$ (c) $0, \pm 2\pi, \pm 4\pi, \ldots$

11. $x = 0 = y, w = z$ **12.** 2, 6 **13.** $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & p^2 \\ p & 1 & -(2p+1) \end{bmatrix}, a = 2p, b = p - 1, c = p$

15. $x = 3, y = -2, a = -9, b = -4$

17. (a) $A^2 = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}, A^3 = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}, A^4 = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$ (b) $A^n = \begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix}$

EXERCISE 11.2

1. (i) –4 (ii) 9 (iii) 12 (iv) 2 (v) 15 (vi) $\frac{4}{9}$ (vii) 0.5 (viii) 4 (ix) 1 (x) $x^2 - 2$ (xi) 0 (xii) 1

2. (i) $\begin{bmatrix} 0 & 1 \\ \frac{1}{4} & \frac{1}{2} \end{bmatrix}$ (ii) $\begin{bmatrix} -1 & -1 \\ 2 & \frac{3}{2} \end{bmatrix}$ (iii) $\begin{bmatrix} -\frac{3}{14} & \frac{1}{7} \\ -\frac{2}{7} & -\frac{1}{7} \end{bmatrix}$ (iv) $\begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & -\frac{1}{4} \end{bmatrix}$ (v) $\begin{bmatrix} 0 & 1 \\ -1 & -4 \end{bmatrix}$ (vi) no inverse

(vii) $\begin{bmatrix} -\frac{1}{5} & \frac{1}{5} \\ -\frac{4}{15} & -\frac{1}{15} \end{bmatrix}$ (viii) $\begin{bmatrix} 1 & -\frac{1}{2} \\ 1 & -\frac{3}{4} \end{bmatrix}$ (ix) $\begin{bmatrix} -4 & -1 \\ 1 & -1 \end{bmatrix}$ (x) $\begin{bmatrix} 1 & 3 \\ 1 & 0 \end{bmatrix}$ (xi) $\begin{bmatrix} 2 & 3 \\ 2 & -3 \end{bmatrix}$ (xii) $\begin{bmatrix} 3 & 3 \\ 1 & -2 \end{bmatrix}$ (xiii) $\begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$

(xiv) $\begin{bmatrix} \frac{3}{x} & -\frac{1}{x} \\ -2 & 1 \end{bmatrix}$ (xv) $\begin{bmatrix} -1 & 1 \\ \frac{3}{x} & -\frac{2}{x} \end{bmatrix}$ (xvi) $\dfrac{x-1}{-x^3 + x^2 + 2x - 1} \begin{bmatrix} \frac{1}{x-1} & 1 - x \\ -x - 1 & x \end{bmatrix}$ **3.** $-2, 2$ **4.** -24 **5.** $-\frac{1}{2}$

6. (a) –29 (b) 80 (c) 0 (d) –11 **7.** (a) $\frac{1}{29} \begin{bmatrix} 5 & -7 & 13 \\ -7 & 4 & 5 \\ 4 & 6 & -7 \end{bmatrix}$ (b) $\frac{1}{80} \begin{bmatrix} 9 & 10 & 7 \\ -1 & -10 & 17 \\ 14 & -20 & 2 \end{bmatrix}$ (c) does not exist.

(d) $\frac{1}{11} \begin{bmatrix} 2 & 3 & -1 \\ 5 & 2 & 3 \\ 4 & 6 & 9 \end{bmatrix}$ **8.** $-5\ \frac{1}{5} \begin{bmatrix} 1 & -2 & 4 \\ -1 & 2 & 1 \\ 2 & 1 & -2 \end{bmatrix}$ **9.** $p = 2\ \frac{1}{35} \begin{bmatrix} 5 & 4 & 7 \\ 10 & -6 & 7 \\ 5 & 11 & -7 \end{bmatrix}$ **11.** $-\frac{23}{4}\ \frac{1}{23} \begin{bmatrix} -15 & 6 & -8 \\ 26 & 8 & 20 \\ -4 & -3 & 4 \end{bmatrix}$

12. $x = 1$

13. $-2\begin{bmatrix} 11 & -9 & -10 \\ -1 & 1 & 1 \\ 8 & -\frac{13}{2} & -\frac{15}{2} \end{bmatrix}$ **15.** (a) $AB = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, $A^{-1} = \frac{1}{5}\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$

(b) $AB = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 3 \\ 1 & -1 & 1 \end{bmatrix}$ (c) $AB = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, $A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 2 \\ 1 & 3 & 2 \end{bmatrix}$

16. (a) 0,1,3 (b) –2,0 **18.** (d) $\alpha = 14, \beta = 3$ (e) 1 (or 0, but we will assume that A is not singular). **21.** (a) –2 (b) $k = -1$

EXERCISE 11.3

1. (i) (1,–1) (ii) (4,–3) (iii) (–3,–3) (iv) (–2,–1) (v) (2,5) (vi) (1,0) (vii) (0,3) (viii) (–2,4)

(ix) (0.5,2) (x) (2,0.6) (xi) (–1,–3) (xii) (–2,2) (xiii) $\left(3, \frac{1}{3}\right)$ (xiv) (5,–2) (xv) (5,4)

2. (i) $a = -2\ b = -4$ (ii) $a = -2\ b \neq -4$ **3.** ±2 **4.** (a) (1,–3,2) (b) (5, –3, –1) (c) (–0.6, –4.2, –1.4)

5. (78,29,–47) **6.** $\left(\frac{4}{11}, \frac{9}{11}, \frac{7}{11}\right)$ **7.** (a) $m \neq -1, 2$ (b) $m = -1$ (c) $m = 2$ **8.** (a) $p \neq 1.2$

(b) $p = 1.2, k \neq 6$ (c) $p = 1.2, k = 6$ **9.** (a) (4,0) (b) Ø (c) (2,–3) (d) (2, 3, –2)
(e) $(4 + 2\lambda, 3 - \lambda, \lambda)$ (f) (1, 2, –2) (g) Ø (h) $(2\lambda - 1, \lambda, \lambda)$ (i) Ø

10. $y = x^2 + 6x - 14$ **11.** $y = -\left(\frac{\lambda}{2} + 6\right)x^2 + \left(\frac{\lambda}{2} + 13\right)x + \lambda$ **12.** (a) i. $\left(-\frac{7\lambda}{11}, -\frac{\lambda}{11}, \lambda\right)$

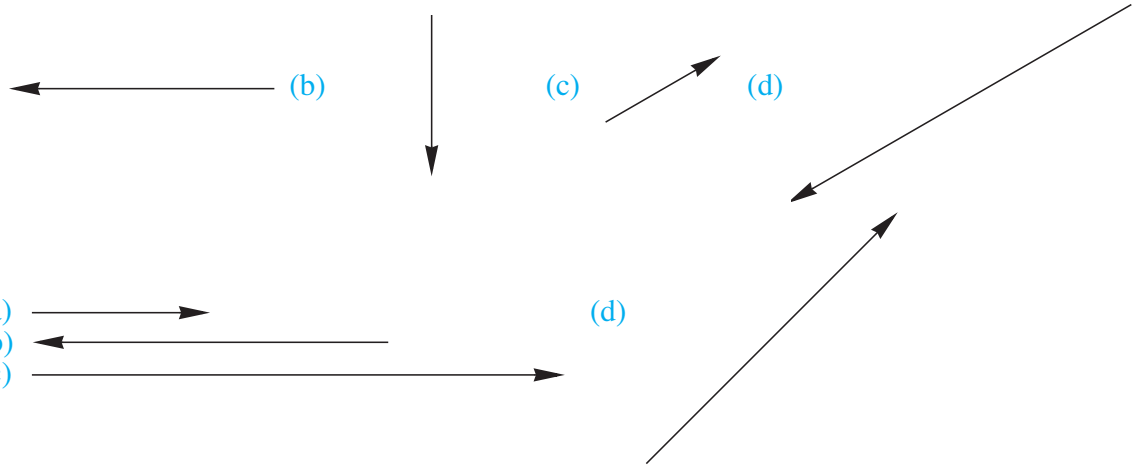
ii. $\left(\frac{7 - 7\lambda}{11}, \frac{1 - \lambda}{11}, \lambda\right)$ (b) (0,0,0) **13.** (a) 1,–2 (b) i. 1,–2 ii. neither 1 nor –2 iii. Ø

EXERCISE 12.1

1. vector **2.** scalar **3.** scalar **4.** vector **5.** vector **6.** vector **7.** scalar **8.** scalar

EXERCISE 12.2

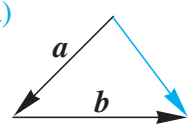
1.(a) (b) (c) (d)

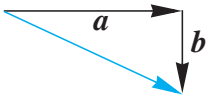


2. (a)
(b)
(c)
(d)

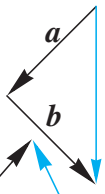
3. (a) {a, b, e, g, u} (b) {a, c}, {d, f}, {b, e} (c) {a,c}, {c, g} (d) {d, f}, {b, e}
(e) {d, f},{b, e},{a, c}

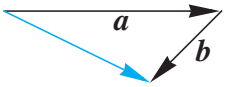
4.

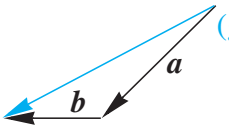
(a) 

(b) 

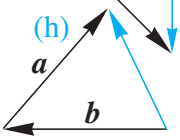
(c) 

(d) 

(e) 

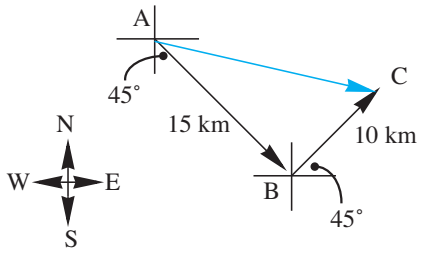
(f) 

(g) **0**

(h) 

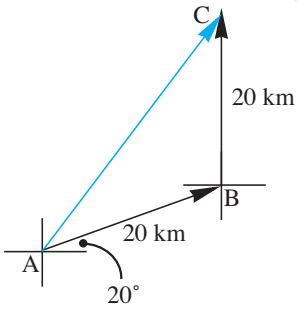
5. (a) **AC** (b) **AB** (c) **AD** (d) **BA** (e) **0** **6.** (a) Y (b) N (c) Y (d) Y (e) N

7. (a) i.



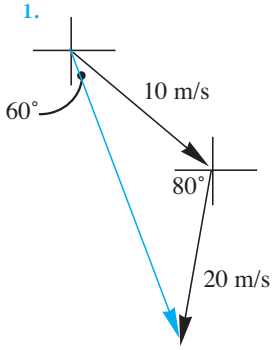
ii. $\sqrt{325}$

(b) i.



ii. $20\sqrt{2(1-\cos 110°)}$

(c) i.



ii. $10\sqrt{5-4\cos 110°}$

8. 72.11 N, E 33°41′ N **9.** 2719 N along river **10.** (b) i. 200 kph N ii. 213.6 kph, N 7°37′ W
11. i. 200 ii. 369.32

EXERCISE 12.3

1. (a) $\boldsymbol{c}-\boldsymbol{a}$ (b) $\boldsymbol{b}-\boldsymbol{c}$ (c) $\frac{1}{2}(\boldsymbol{b}+\boldsymbol{a})$ **2.** (a) $\boldsymbol{b}-\boldsymbol{a}$ (b) $\boldsymbol{b}-2\boldsymbol{a}$ (c) $2\boldsymbol{b}-3\boldsymbol{a}$ (d) $\frac{1}{2}(\boldsymbol{b}+2\boldsymbol{a})$

3. (a) **0** (b) **PS** (c) **AY** (d) 6**OC** **4.** (a) $\frac{1}{2}(\boldsymbol{b}+\boldsymbol{a})$ (b) $\frac{1}{3}(2\boldsymbol{b}+\boldsymbol{a})$ (c) $\frac{1}{4}(\boldsymbol{a}+\boldsymbol{b}+2\boldsymbol{c})$

7. (a) $\boldsymbol{c}-\boldsymbol{b}$ (b) $\boldsymbol{c}+\boldsymbol{a}$ (c) $\boldsymbol{a}+\boldsymbol{c}-2\boldsymbol{b}$ **8.** (a) $2\sqrt{21}$ (b) $2\sqrt{26}$ **15.** $m=\frac{13}{23}, n=\frac{50}{23}$ **16.** $m=\frac{4}{3}$

EXERCISE 12.4

1. (i) $4\boldsymbol{i}+28\boldsymbol{j}-4\boldsymbol{k}$ (ii) $12\boldsymbol{i}+21\boldsymbol{j}+15\boldsymbol{k}$ (iii) $-2\boldsymbol{i}+7\boldsymbol{j}-7\boldsymbol{k}$ (iv) $-6\boldsymbol{i}-12\boldsymbol{k}$

2. (i) $3\boldsymbol{i}-4\boldsymbol{j}+2\boldsymbol{k}$ (ii) $-8\boldsymbol{i}+24\boldsymbol{j}+13\boldsymbol{k}$ (iii) $18\boldsymbol{i}-32\boldsymbol{j}+\boldsymbol{k}$ (iv) $-15\boldsymbol{i}+36\boldsymbol{j}+12\boldsymbol{k}$

3. (i) $\begin{pmatrix}11\\0\\8\end{pmatrix}$ (ii) $\begin{pmatrix}-27\\1\\-22\end{pmatrix}$ (iii) $\begin{pmatrix}-3\\-6\\12\end{pmatrix}$ (iv) $\begin{pmatrix}16\\-1\\14\end{pmatrix}$ **4.** $\begin{pmatrix}-5\\3\end{pmatrix}$ **5.** $\begin{pmatrix}-2\\3\end{pmatrix}$, (–2,3)

6. (i) $8\boldsymbol{i}-4\boldsymbol{j}-28\boldsymbol{k}$ (ii) $-19\boldsymbol{i}-7\boldsymbol{j}-16\boldsymbol{k}$ (iii) $-17\boldsymbol{i}+\boldsymbol{j}+22\boldsymbol{k}$ (iv) $40\boldsymbol{i}+4\boldsymbol{j}-20\boldsymbol{k}$

7. (i) $\begin{pmatrix}20\\1\\25\end{pmatrix}$ (ii) $\begin{pmatrix}12\\2\\16\end{pmatrix}$ (iii) $\begin{pmatrix}-4\\-38\\-32\end{pmatrix}$ (iv) $\begin{pmatrix}-20\\-22\\-40\end{pmatrix}$ **8.** A = –4, B = –7

9. i. (2,–5) ii. (–4,3) iii. (–6,–5) **10.** Depends on basis used. Here we used: East as $\boldsymbol{i}$, North $\boldsymbol{j}$ and vertically up $\boldsymbol{k}$ (b) $D = 600\boldsymbol{i}-800\boldsymbol{j}+60\boldsymbol{k}$, $A = -1200\boldsymbol{i}-300\boldsymbol{j}+60\boldsymbol{k}$ (c) $1800\boldsymbol{i}-500\boldsymbol{j}$

EXERCISE 12.5

1. (i) $\sqrt{10}$ (ii) $5\sqrt{2}$ (iii) $\sqrt{30}$ (iv) 3 (v) $\sqrt{53}$ (vi) $\sqrt{41}$ (vii) $\sqrt{14}$ (viii) $\sqrt{17}$

2. (i) $\frac{1}{\sqrt{2}}(\boldsymbol{i}+\boldsymbol{j})$ (ii) $\frac{1}{\sqrt{41}}(4\boldsymbol{i}+5\boldsymbol{j})$ (iii) $\frac{1}{\sqrt{5}}(-\boldsymbol{i}-2\boldsymbol{j})$ (iv) $\frac{1}{\sqrt{46}}(\boldsymbol{i}+6\boldsymbol{j}-3\boldsymbol{k})$ (v) $\frac{1}{\sqrt{5}}(\boldsymbol{i}+2\boldsymbol{k})$

(vi) $\frac{1}{\sqrt{17}}(2\boldsymbol{i}-2\boldsymbol{j}-3\boldsymbol{k})$ (vii) $\frac{1}{3}\begin{pmatrix}2\\1\\2\end{pmatrix}$ (viii) $\frac{1}{3\sqrt{3}}\begin{pmatrix}-1\\5\\1\end{pmatrix}$ **3.** i. Depends on the basis: $-3\boldsymbol{i}+4\boldsymbol{j}+\boldsymbol{k}$

or $-4\boldsymbol{i}-3\boldsymbol{j}+\boldsymbol{k}$ ii. $\sqrt{26}$ **4.** (a) $\sqrt{3}(\boldsymbol{i}-\boldsymbol{j}+\boldsymbol{k})$ (b) $\frac{1}{4}(3\boldsymbol{i}-\boldsymbol{j}+\sqrt{2}\boldsymbol{k})$ **5.** $\pm\sqrt{11}$ **6.** $\sqrt{13}$

EXERCISE 12.6

1. (a) 4 (b) –11.49 (c) 25 **2.** (i) 12 (ii) 27 (iii) –8 (iv) –49 (v) 4 (vi) –21 (vii) 6 (viii) –4
(ix) –10 **3.** (i) 79° (ii) 108° (iii) 55° (iv) 50° (v) 74° (vi) 172° (vii) 80° (viii) 58° **4.** (i) –8 (ii) 0.5

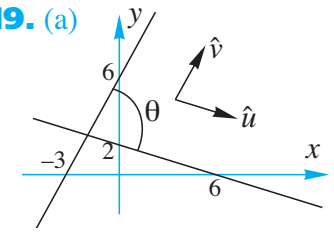
5. (a) –6 (b) 2 (c) not possible (d) 5 (e) not possible (f) 0 **6.** (a) $4-2\sqrt{3}$ (b) $2\sqrt{3}-4$

(c) $14-2\sqrt{3}$ (d) not possible **7.** 1 **8.** 105.2° **9.** $x=-\frac{16}{7}, y=-\frac{44}{7}$ **10.** $\pm\frac{1}{\sqrt{11}}(-\boldsymbol{i}+\boldsymbol{j}+3\boldsymbol{k})$

12. (a) $\lambda(-16\boldsymbol{i}-10\boldsymbol{j}+\boldsymbol{k})$ (b) e.g. $\boldsymbol{i}+\boldsymbol{j}+\frac{3}{7}\boldsymbol{k}$ **14.** $\boldsymbol{a}\perp\boldsymbol{b}-\boldsymbol{c}$ if $\boldsymbol{b}\neq\boldsymbol{c}$ or $\boldsymbol{b}=\boldsymbol{c}$ **15.** (a) $\left(\frac{3}{5},\frac{4}{5}\right)$

(b) $\left(\frac{\sqrt{2}}{2},\frac{1}{2},-\frac{1}{2}\right)$ **16.** (a) $\left(-\frac{2}{3},\frac{2}{3},\frac{1}{3}\right)$ (b) 131.8°, 48.2°, 70.5° **18.** (a) $\frac{1}{3}$ (b) $\frac{1}{\sqrt{3}}$

19. (a)



(b) $\hat{\boldsymbol{u}}=\frac{1}{\sqrt{10}}(3\boldsymbol{i}-\boldsymbol{j})$, $\hat{\boldsymbol{v}}=\frac{1}{\sqrt{5}}(\boldsymbol{i}+2\boldsymbol{j})$ **20.** $\frac{1}{2}(-\boldsymbol{i}+\boldsymbol{j}+\sqrt{2}\boldsymbol{k})$
(c) 81.87°

25. (a) Use $\boldsymbol{i}$ as a 1 km eastward vector and $\boldsymbol{j}$ as a 1 km northwards vector (b) $\overrightarrow{WD}=4\boldsymbol{i}+8\boldsymbol{j}$,
$\overrightarrow{WS}=13\boldsymbol{i}+\boldsymbol{j}$ and $\overrightarrow{DS}=9\boldsymbol{i}-7\boldsymbol{j}$ (c) $\frac{1}{\sqrt{80}}(4\boldsymbol{i}+8\boldsymbol{j})$ (d) $\frac{d}{\sqrt{80}}(4\boldsymbol{i}+8\boldsymbol{j})$ (e) $3\boldsymbol{i}+6\boldsymbol{j}$

EXERCISE 12.7.1

1. (a) i. $r=\boldsymbol{i}+2\boldsymbol{j}$ ii. $r=-5\boldsymbol{i}+11\boldsymbol{j}$ iii. $r=5\boldsymbol{i}-4\boldsymbol{j}$ (b) line joins (1,2) & (5,–4)
2. (a) $r=2\boldsymbol{i}+5\boldsymbol{j}+\lambda(3\boldsymbol{i}-4\boldsymbol{j})$ (b) $r=-3\boldsymbol{i}+4\boldsymbol{j}+\lambda(-\boldsymbol{i}+5\boldsymbol{j})$ (c) $r=\boldsymbol{j}+\lambda(7\boldsymbol{i}+8\boldsymbol{j})$

(d) $r=\boldsymbol{i}-6\boldsymbol{j}+\lambda(2\boldsymbol{i}+3\boldsymbol{j})$ (e) $\boldsymbol{r}=\begin{pmatrix}-1\\-1\end{pmatrix}+\lambda\begin{pmatrix}-2\\10\end{pmatrix}$ or $r=-\boldsymbol{i}-\boldsymbol{j}+\lambda(-2\boldsymbol{i}+10\boldsymbol{j})$

(f) $r=\begin{pmatrix}1\\2\end{pmatrix}+\lambda\begin{pmatrix}5\\1\end{pmatrix}$ or $r=\boldsymbol{i}+2\boldsymbol{j}+\lambda(5\boldsymbol{i}+\boldsymbol{j})$

3. (a) $r=2\boldsymbol{i}+3\boldsymbol{j}+\lambda(2\boldsymbol{i}+5\boldsymbol{j})$ (b) $r=\boldsymbol{i}+5\boldsymbol{j}+\lambda(-3\boldsymbol{i}-4\boldsymbol{j})$ (c) $r=4\boldsymbol{i}-3\boldsymbol{j}+\lambda(-5\boldsymbol{i}+\boldsymbol{j})$
4. (a) $r=9\boldsymbol{i}+5\boldsymbol{j}+\lambda(\boldsymbol{i}-3\boldsymbol{j})$ (b) $r=6\boldsymbol{i}-6\boldsymbol{j}+t(-4\boldsymbol{i}-2\boldsymbol{j})$ (c) $r=-\boldsymbol{i}+3\boldsymbol{j}+\lambda(-4\boldsymbol{i}+8\boldsymbol{j})$

(d) $r=\boldsymbol{i}+2\boldsymbol{j}+\mu\left(\frac{1}{2}\boldsymbol{i}-\frac{1}{3}\boldsymbol{j}\right)$ **5.** (a) $\begin{matrix}x=-8+2\mu\\y=10+\mu\end{matrix}$ (b) $\begin{matrix}x=7-3\mu\\y=4-2\mu\end{matrix}$ (c) $\begin{matrix}x=5+2.5\mu\\y=3+0.5\mu\end{matrix}$

(d) $\begin{aligned} x &= 0.5 - 0.1t \\ y &= 0.4 + 0.2t \end{aligned}$ **6.** (a) $\frac{x-1}{3} = y - 3$ (b) $\frac{x-2}{-7} = \frac{y-4}{-5}$ (c) $x + 2 = \frac{y+4}{8}$

(d) $x - 0.5 = \frac{y - 0.2}{-11}$ (e) $x = 7$ (f) $y = 6$ **7.** (a) $r = 2\boldsymbol{j} + t(3\boldsymbol{i} + \boldsymbol{j})$ (b) $r = 5\boldsymbol{i} + t(\boldsymbol{i} + \boldsymbol{j})$

(c) $\boldsymbol{r} = -6\boldsymbol{i} + t(2\boldsymbol{i} + \boldsymbol{j})$ **8.** (a) $6\boldsymbol{i} + 13\boldsymbol{j}$ (b) $-\frac{16}{3}\boldsymbol{i} - \frac{28}{3}\boldsymbol{j}$ **9.** $r = 2\boldsymbol{i} + 7\boldsymbol{j} + t(4\boldsymbol{i} + 3\boldsymbol{j})$

11. (a) (4,–2), (–1,1), (9,–5) (b) –2 (d) $r = 4\boldsymbol{i} - 2\boldsymbol{j} + \lambda(-5\boldsymbol{i} + 3\boldsymbol{j})$ (e) i. $M \parallel L$ ii. $M = L$

12. $4x + 3y = 11$ **13.** (a) $\frac{-3}{\sqrt{13}}, \frac{2}{\sqrt{13}}$ (b) $\frac{4}{5}, \frac{3}{5}$ **14.** (b) ii. & iii. **15.** (–83,–215)

16. $r = \frac{k}{7}(19\boldsymbol{i} + 20\boldsymbol{j})$ **17.** (a) $\left(\frac{92}{11}, \frac{31}{11}\right)$ (b) Ø (c) Lines are coincident, all points are common.

EXERCISE 12.7.2

1. (a) No (b) 52.5 mins after A **2.** (a) i. $\boldsymbol{r}_A = \begin{pmatrix} 5 \\ -1 \end{pmatrix} + t\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ ii. $\boldsymbol{r}_B = \begin{pmatrix} 4 \\ 5 \end{pmatrix} + t\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ (b) No

(c) i. $\begin{pmatrix} 4 \\ 5 \end{pmatrix} + (t-1)\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ ii. 11 am

EXERCISE 12.7.3

1. (a) $r = 2\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k} + t(\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k})$ (b) $r = 2\boldsymbol{i} - 3\boldsymbol{j} - \boldsymbol{k} + t(-2\boldsymbol{i} + \boldsymbol{k})$

2. (a) $r = 2\boldsymbol{i} + 5\boldsymbol{k} + t(\boldsymbol{i} + 4\boldsymbol{j} + 3\boldsymbol{k})$ (b) $r = 3\boldsymbol{i} - 4\boldsymbol{j} + 7\boldsymbol{k} + t(4\boldsymbol{i} + 9\boldsymbol{j} - 5\boldsymbol{k})$

(c) $r = 4\boldsymbol{i} + 4\boldsymbol{j} + 4\boldsymbol{k} + t(7\boldsymbol{i} + 7\boldsymbol{k})$ **3.** (a) $\frac{x}{3} = \frac{y-2}{4} = \frac{z-3}{5}$ (b) $\frac{x+2}{5} = \frac{z+1}{-2}, y = 3$

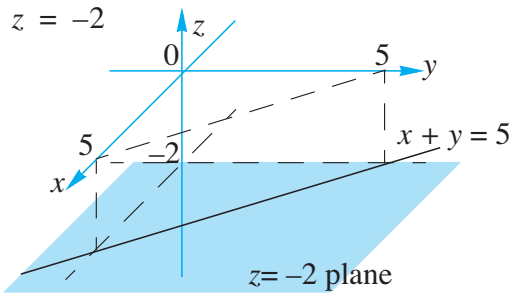
(c) $x = y = z$ **4.** $\begin{aligned} x &= 5 - 7t \\ y &= 2 + 2t \\ z &= 6 - 4t \end{aligned}$ $\boldsymbol{r} = \begin{pmatrix} 5 \\ 2 \\ 6 \end{pmatrix} + t\begin{pmatrix} -7 \\ 2 \\ -4 \end{pmatrix}$ $\frac{x-5}{-7} = \frac{y-2}{2} = \frac{z-6}{-4}$ **5.** $\left(\frac{13}{5}, \frac{23}{5}, 0\right)$

6. (a) $\begin{aligned} x &= 2 + 3t \\ y &= 5 + t \\ z &= 4 + 0.5t \end{aligned}$ (b) $\begin{aligned} x &= 1 + 1.5t \\ y &= t \\ z &= 4 - 2t \end{aligned}$ (c) $\begin{aligned} x &= 3 - t \\ y &= 2 - 3t \\ z &= 4 + 2t \end{aligned}$ (d) $\begin{aligned} x &= 1 + 2t \\ y &= 3 + 2t \\ z &= 2 + 0.5t \end{aligned}$

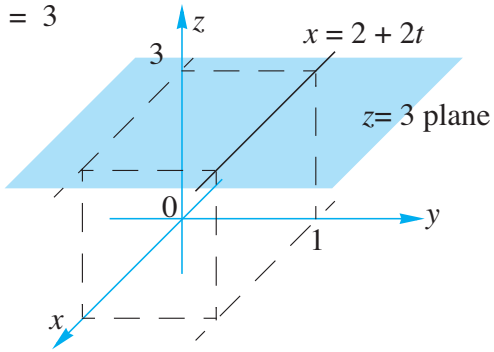
7. (a) $\frac{x-4}{3} = \frac{y-1}{-4} = \frac{z+2}{-2}$ (b) $x = 2, y = \frac{z-1}{-3}$ **9.** (a) $\frac{x+1}{2} = y - 3 = \frac{z-5}{-1}$

(b) $\frac{x-2}{2} = \frac{z-1}{-2}, y = -1$ **10.** (a) $(1, -1, 0)$ (b) $a = 15, b = -11$

11. (a) $\begin{aligned} x &= 1 + t \\ y &= 4 - t \\ z &= -2 \end{aligned}$



(b) $\begin{aligned} x &= 2 + 2t \\ y &= 1 \\ z &= 3 \end{aligned}$



12. $\boldsymbol{r} = \begin{pmatrix} 1 \\ 0.5 \\ 2 \end{pmatrix} + t\begin{pmatrix} 2 \\ -1.5 \\ 1 \end{pmatrix}$. Line passes through (1, 0.5, 2) and is parallel to the vector $2\boldsymbol{i} - \frac{3}{2}\boldsymbol{j} + \boldsymbol{k}$

13. (a) 54.74° (b) 82.25° (c) 57.69° **14.** (a) (4, 10.5, 15) (b) Does not intersect.

15. (a) L: $x = \frac{y-2}{2} = \frac{z}{5}$, M: $\frac{x+1}{2} = \frac{y+1}{3} = \frac{z-1}{-2}$ (b) Ø (c) 84.92° (d) i. (0, 2, 0) ii. $\left(0, \frac{1}{2}, 0\right)$

18. $\frac{x}{4} = \frac{y}{9} = \frac{z}{3}$ **19.** $k = -\frac{7}{2}$ **20.** 64° **21.** 3 or –2 **22.** $12\boldsymbol{i} + 6\boldsymbol{j} - 7\boldsymbol{k}$ (or any multiple thereof) **23.** Not parallel. Do not intersect. Lines are skew.

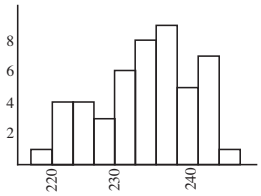
EXERCISE 13.1

1. (a) i. 14 500 ii. 2 000 (b) 305 (304.5) **2.** Sample size is large but may be biassed by factors such as the location of the catch. Population estimate is 5 000. **3.** (a) i. 1500 ii. 120 (b) 100 (c) 1 000 **4.** (a), (c) numerical, (b), (d), (e) categorical **5.** (a), (d) discrete, (b), (c), (e) continuous

EXERCISE 13.2

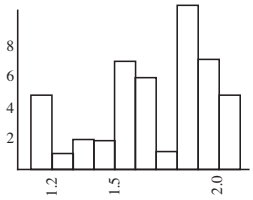
1.

218-220	221-223	224-226	227-229	230-232	233-235	236-238	239-241	242-244	245-247
1	4	4	3	6	8	9	5	7	1



2.

1.1-1.2	1.2-1.3	1.3-1.4	1.4-1.5	1.5-1.6	1.6-1.7	1.7-1.8	1.8-1.9	1.9-2.0	2.0-2.1
5	1	2	2	7	6	1	12	7	5



3. Set A Mode = 29.1 Mean = 27.2 Median = 27.85
Set B Mode = 9 Mean = 26.6 Median = 9. Set B is much more spread out than set A and although the two sets have a similar mean, they have very different mode and median.

EXERCISE 13.3

1. Mode = 236-238gms Mean = 234gms Median = 235gms **2.** Mode = 1.8-1.9gms Mean = 1.69gms Median = 1.80gms **3.** Set A Mode = 29.1 Mean = 27.2 Median = 27.85 Set B Mode = 9 Mean = 26.6 Median = 9.**4.** (a) $27522 (b) $21025 (c) Median **5.** i. $233 300 ii. $169 000 iii. Median **6.** (a) 14.375 (b) 14.354

EXERCISE 13.4

1. (a) Sample A Mean = 1.99kg Sample B Mean = 2.00kg (b) Sample A Sample std = 0.0552kg Sample B Sample std = 0.1877kg (c) Sample A Population std = 0.0547kg Sample B Population std = 0.1858kg **2.** (a) 16.41 (b) 6.84 **3.** Mean = 49.97 Std = 1.365

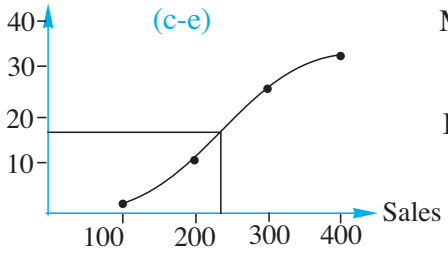
EXERCISE 13.5

1. (a) Med = 5, Q1 = 2, Q3 = 7, IQR = 5 (b) Med = 3.3, Q1 = 2.8, Q3 = 5.1, IQR = 2.3 (c) Med = 163.5, Q1 = 143, Q3 =182, IQR = 39 (d) Med = 1.055, Q1 = 0.46, Q3 = 1.67, IQR = 1.21 (e) Med = 5143.5, Q1 = 2046, Q3 = 6252, IQR = 4206
2. (a) Med = 3, Q1 = 2, Q3 = 4, IQR = 2 (b) Med = 13, Q1 = 12, Q3 = 13, IQR = 1 (c) Med = 2, Q1 = 2, Q3 = 2.5, IQR = 0.5 (d) Med = 40, Q1 = 30, Q3 = 50, IQR = 20

(e) Med = 20, Q1 = 15, Q3 = 22.5, IQR = 7.5

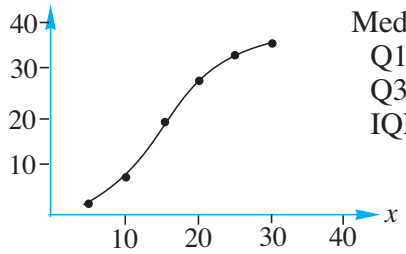
3. (a) \$84.67 (b) \$147.85 (c) \$11 (d) Q1 = \$4.50, Q3 = \$65 IQR = \$60.50 (e) Median & IQR.

4. (a) 2.35 (b) 1.25 (c) 2 (d) Q1 = 1, Q3 = 3, IQR = 2 **5.** (a) \$232 (b) \$83



Med = \$220
Q1 = \$160
Q3 = \$310
IQR = \$150

6.



Med = 14
Q1 = 10
Q3 = 19
IQR = 9

EXERCISE 13.6

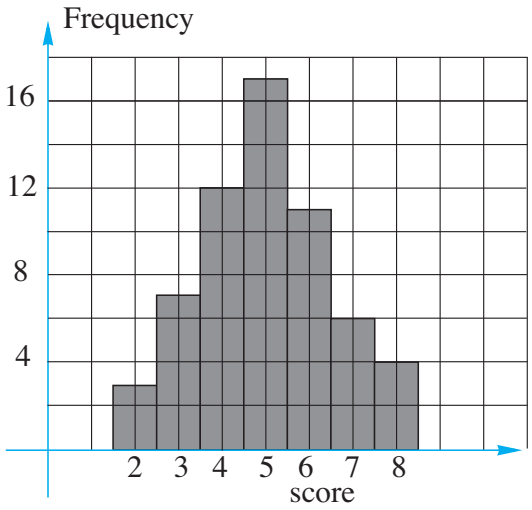
1. (a) Sample–100 randomly selected patients, population – all suffering from AIDS

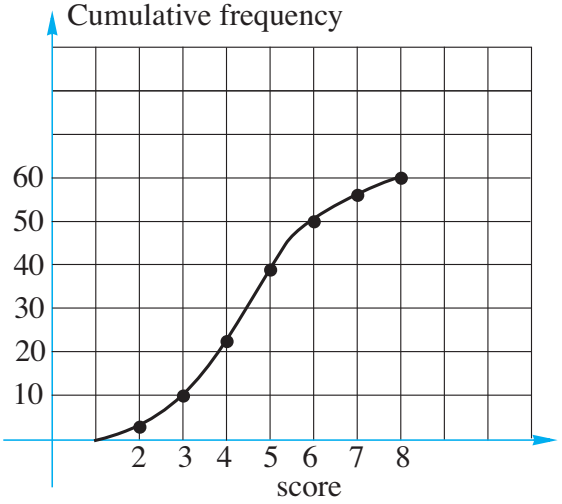
(b) Sample–1000 working aged people in N.S.W, population – all working aged people in N.S.W

(c) Sample – John's I.B Higher Maths class, population – all seniors at Nappa Valley High School.

2. Discrete: a, b, d; Continuous: c, e, f, g.

3. (b)





4. suggested answers only: (a) 200–224; 225–249; 250–274; . . . 575–599

(b) 100–119; 120–139; . . . 400–419 (c) 440–459; 460–479; . . . 780–799.

5. Make use of your graphics calculator

6. (a) 16 (b) graphics calculator (c) 15.23

(d) 15.5 (e) Q1 = 14, Q3 = 17 (f) 15.87 (2dec.pl.)

7. (a) 30–34 (b) graphics calculator (c) 30.4

(d) 32 (approx)

8. (b) 215.5 (c) 216.2 **9.** 48.17

10. (a) Q1~ 35, Q3~ 95 (b) ~ 104 (c) 60% (d) 67.15

11. range = 19, s = 5.49 **12.** 5.8; 1.50 **13.** 17.4; $s_n = 3.12$ $s_{n-1} = 3.18$

14. (a) 6 (b) 7 (c) Q1 = 5, Q3 = 7

(d) 2 (e) 6.15 (f) 1.61

15. $s_n = 18.8$, $s_{n-1} = 19.1$ **16.** 14.18

EXERCISE 14.1

1. 15 **2.** (a) 25 (b) 625 **3.** (a) 24 (b) 256 **4.** (a) 24 (b) 48 **5.** 15 **6.** 270 **7.** 120 **8.** 336 **9.** 60
10. (a) 362880 (b) 80640 (c) 1728 **11.** 20 **12.** (a) 10! (b) 2×8! (c) i. 2×9! ii. 8×9! **13.** 34650
14. 4200 **15.** 4

EXERCISE 14.2

1. 792 **2.** (a) 1140 (b) 171 **3.** 1050 **4.** 70 **5.** 2688 **6.** (a) 210 (b) 420 **7.** 24000 **8.** 8 **9.** 155 **10.** 5

EXERCISE 14.3

1. (a) 120 (b) 325 **2.** 5040 **3.** 5040; (a)144 (b) 1440 **4.** (a) 720 (b) 240 **5.** 11760 **6.** 7056; 4606 **7.** (a) 840 (b) 1680 **8.** 190 **9.** 10080 **10.** 226800 **11.** (a) 715 (b) 315 (c) 665 **13.** ${}^{n}C_{2}$ **14.** ${}^{n}C_{4}$ **15.** (b) 92 **16.** 252 **17.** (a) 1287 (b) 560 **18.** 256 **19.** 288 **20.** (a) 10080 (b) 30240 (c) 14400 **21.** 10080, 1080 **22.** 3528000 **23.** 720; 240 **24.** 103680 **25.** (a) 12 (b) 128 **26.** 2880 **27.** (a) 30030 (b) 37310 **28.** 77055 **29.** (a) 48 (b) 72

EXERCISE 15.1

1. (a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{2}{5}$ **2.** (a) $\frac{2}{7}$ (b) $\frac{5}{7}$ **3.** (a) $\frac{5}{26}$ (b) $\frac{21}{26}$ **4.** {HH, HT, TH, TT} (a) $\frac{1}{4}$ (b) $\frac{3}{4}$

5. {HHH,HHT,HTH,THH,TTT,TTH,THT,HTT} (a) $\frac{3}{8}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ **6.** (a) $\frac{2}{9}$ (b) $\frac{2}{9}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

7. (a) $\frac{1}{2}$ (b) $\frac{3}{10}$ (c) $\frac{9}{20}$ **8.** (a) $\frac{11}{36}$ (b) $\frac{1}{18}$ (c) $\frac{1}{6}$ (d) $\frac{5}{36}$ **9.** {GGG, GGB. GBG, BGG, BBB, BBG,

BGB, GBB} (a) $\frac{1}{8}$ (b) $\frac{3}{8}$ (c) $\frac{1}{2}$ **10.** (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{4}$ **11.** (a) $\frac{3}{8}$ (b) $\frac{1}{4}$ (c) $\frac{3}{8}$ (d) $\frac{3}{4}$

12. (a){(1, H),(2, H),(3, H),(4,H),(5, H),(6, H),(1, T),(2, T),(3, T),(4, T),(5, T),(6,T)}

(b) $\frac{1}{4}$ **13.** (a) $\frac{1}{216}$ (b) $\frac{1}{8}$ (c) $\frac{3}{8}$

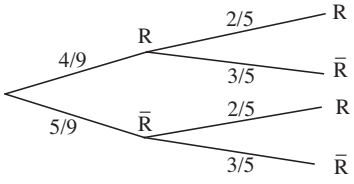
EXERCISE 15.2

1. (a) $\frac{1}{4}$ (b) $\frac{5}{8}$ (c) $\frac{3}{4}$ **2.** (a) $\frac{1}{13}$ (b) $\frac{1}{2}$ (c) $\frac{1}{26}$ (d) $\frac{7}{13}$ **3.** $\frac{9}{26}$ **4.** (a) 1.0 (b) 0.3 (c) 0.5 **5.** (a) 0.65 (b) 0.70 (c) 0.65 **6.** (a) 0.95 (b) 0.05 (c) 0.80 **7.** (a) {TTT,TTH,THT,HTT,HHH,HHT,HTH, THH} (b) $\frac{3}{8}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$ (e) $\frac{3}{8}$ **8.** (a) $\frac{6}{25}$ (b) $\frac{6}{25}$ (c) $\frac{13}{25}$ **9.** (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{6}$ (e) $\frac{7}{13}$

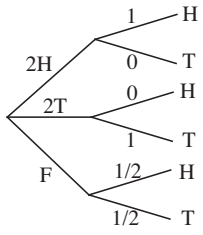
10. (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{8}{13}$ (d) $\frac{7}{13}$ **11.** (a) 0.1399 (b) 0.8797 (c) 0.6 **12.** (b) $\frac{4}{15}$ (c) $\frac{4}{15}$ (d) $\frac{11}{15}$

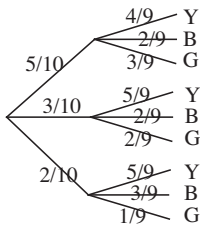
EXERCISE 15.3

1. (a) 0.7 (b) 0.75 (c) 0.50 (d) 0.5 **2.** (a) 0.5 (b) 0.83 (c) 0.10 (d) 0.90

3. (a) 

(b) $\frac{8}{45}$ (c) $\frac{22}{45}$ (d) $\frac{6}{11}$ **4.** (a) 0.5 (b) 0.30 (c) 0.25

5. (a) 

(b) $\frac{1}{2}$ (c) $\frac{2}{3}$ **6.** $\frac{1}{3}$ **7.** (a) 

(b) $\frac{31}{45}$ (c) $\frac{2}{9}$ **8.** $\frac{2}{3}$

9. (a) 0.88 (b) 0.42 (c) 0.6 (d) 0.28 **10.** (a) 0.33 (b) 0.49 (c) 0.82 (d) 0.551 **11.** (a) 0.22 (b) 0.985

(c) 0.8629 **12.** (a) 0.44 (b) 0.733 **14.** (a) 0.512 (b) 0.128 (c) 0.8571 **15.** (a) 0.2625 (b) 0.75 (c) 0.4875 (d) 0.7123 **16.** (a) 0.027 (b) 0.441 (c) 0.453

EXERCISE 15.4

1. (a) 0.042 (b) 0.7143 **2.** (a) 0.4667 (b) 0.3868 **3.** (a) $\frac{5}{7}$ (b) $\frac{9}{13}$ **4.** $\frac{5}{9}$ **5.** (b) i. $\frac{1}{40}$ ii. 0.2

6. (a) i. $\frac{2N-m}{2N}$ ii. $\frac{2(N-m)}{2N-m}$ (b) $\frac{m}{m+(N-m)2^n}$ **7.** $\frac{9}{19}$ **8.** a. 0.07 b. 0.3429 c. 0.30 d. 0.0282 **9.** a. 0.8008 b. 0.9767 c. 0.0003 **10.** a. 0.0464 b. 0.5819 c. 0.9969

11. a. 0.2 b. 0.08 c. 0.72 d. $\begin{bmatrix}0.8 & 0.2\\ 0.4 & 0.6\end{bmatrix}$; $\begin{bmatrix}0.72 & 0.28\\ 0.56 & 0.44\end{bmatrix}$ g. 0.688

EXERCISE 15.5

1. (a) $\frac{5}{126}$ (b) $\frac{5}{18}$ (c) $\frac{1}{126}$ **2.** (a) $\frac{1}{5}$ (b) $\frac{1}{10}$ (c) $\frac{2}{5}$ (d) $\frac{3}{5}$ **3.** (a) $\frac{72}{5525}$ (b) $\frac{1}{5525}$ (c) $\frac{1}{1201}$ **4.** $\frac{2}{5}$

5. (a) $\frac{63}{143}$ (b) $\frac{133}{143}$ **6.** (a) $\frac{5}{12}$ (b) $\frac{5}{33}$ (c) $\frac{5}{6}$ **7.** $\frac{3}{11}$ **8.** (a) $\frac{4}{13}$ (b) $\frac{9}{13}$ **9.** (a) $\frac{67}{91}$ (b) $\frac{22}{91}$

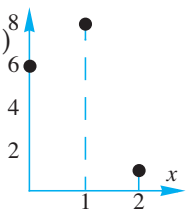
10. (a) $\frac{1}{4}$ (b) $\frac{1}{28}$ (c) $\frac{5}{14}$ **11.** (a) $\frac{5}{28}$ (b) $\frac{1}{28}$ **12.** $\frac{6}{13}$ **13.** (a) $\frac{1}{6}$ (b) $\frac{1}{4}$ **14.** (a) $\frac{1}{210}$ (b) $\frac{7}{9}$

15. (a) $\frac{7}{1938}$ (b) 0.6 **16.** $\frac{11}{21}$

EXERCISE 16.1

1. 0.3 **2.** (a) 0.1 (b) i. 0.2 ii. 0.7

3. (a) $p(0)=\frac{6}{15}, p(1)=\frac{8}{15}, p(2)=\frac{1}{15}$ (b) $15\cdot p(x)$



(c) $\frac{14}{15}$

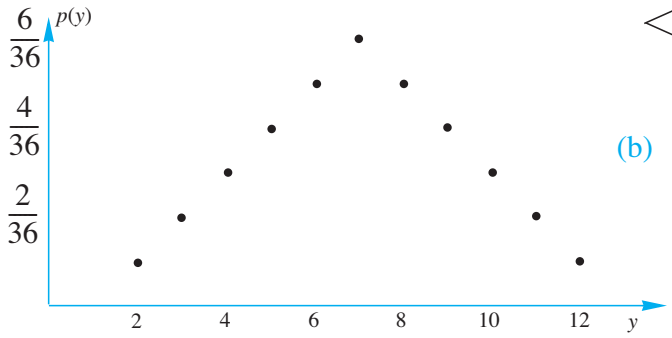
4. (a) {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

(b)

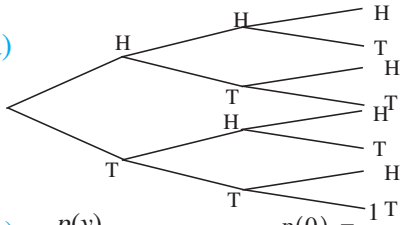
x	2	3	4	5	6	7	8	9	10	11	12
$p(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

(c) $\frac{5}{36}$

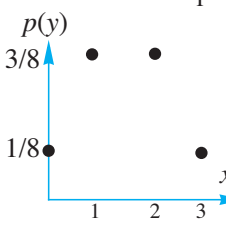
(d)



5. (a)



(b)



$p(0)=\frac{1}{8}$

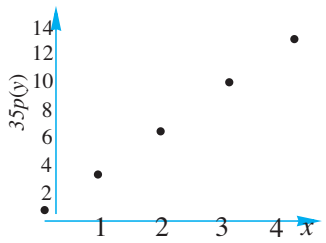
$p(1)=\frac{3}{8}$

$p(2)=\frac{3}{8}$

$p(3)=\frac{1}{8}$

(c) $\frac{4}{7}$

6. (a) $\frac{1}{35}$ (b) i. $p(0) = \frac{1}{35}, p(1) = \frac{4}{35}, p(2) = \frac{7}{35}$ $p(3) = \frac{10}{35}, p(4) = \frac{13}{35}$ ii.



(c) $\frac{6}{7}$ **7.** (a) i. 0.9048 ii. 0.09048 (b) 0.0002 **8.** 0.3712 **9.** $p(0) = \frac{11}{30}, p(-1) = \frac{1}{2}, p(3) = \frac{2}{15}$ (b) i. $\frac{11}{30}$ ii. $\frac{13}{15}$

10.

n	0	1	2
$P(N=n)$	$\frac{6}{15}$	$\frac{8}{15}$	$\frac{1}{15}$

11. (a)

n	1	2	3	4
$P(N=n)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

(b)

s	2	3	4	5	6	7	8
$P(S=s)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

12. (a) 0.81 (b) 0.2439

EXERCISE 16.2

1. (a) 2.8 (b) 1.86 **2.** (a) 3 (b) i. 1 ii. 1 (c) i. 6 ii. 0.4 **3.** (a) i. 1.3 ii. 2.5 iii. –0.1 (b) i. 0.9 ii. 7.29 (c) i. $\frac{31}{60}$ ii. 0.3222 **4.** $\mu = \frac{2}{3}, \sigma^2 = 0.3556$ **5.** (a) 7 (b) 5.8333 **6.** $np = 3 \times \frac{1}{2} = 1.5$ **7.** (a) $\frac{1}{25}$ (b) 2.8 (c) 1.166 **8.** (a) 0.1 (b) i. 0.3 ii. 1 (c) i. 0 ii. 1 iii. 2 **9.** 5.56 **10.** $p(0) = \frac{35}{120}, p(1) = \frac{63}{120}, p(2) = \frac{21}{120}, p(3) = \frac{1}{120}$ (b) i. 0.9 ii. 0.49 (c) $W = 3N - 3$ $E(W) = -0.3$ **11.** (a) \$–1.00 (b) both the same **12.** (a) 50 (b) 18 (c) 2 **13.** (a) 11 (b) $\frac{\sqrt{3}}{3}$ (c) –4 **14.** (a) 0.75 (b) 0.6339 **15.** (a) $E(X) = 1 - 2p$, $\text{Var}(X) = 4p(1-p)$ (b) i. $n(1-2p)$ ii. $4np(1-p)$

16.

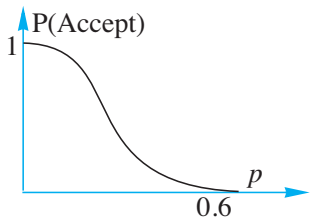
n	0	1	2
$P(N=n)$	$\frac{28}{45}$	$\frac{16}{45}$	$\frac{1}{45}$

W = 21.43 **17.** (a) $a = \frac{2}{3}, 0 \le b \le 1$ (b) $E(X) = \frac{b+1}{3}, Var(X) = \frac{1}{9}(2 + 7b - b^2)$ **18.** (a) $E(X) = 4, \text{Var}(X) = 20$

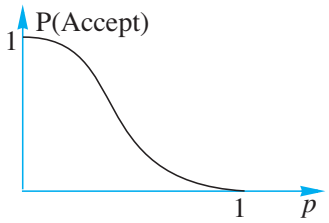
EXERCISE 16.3

1. (a) 0.2322 (b) 0.1737 (c) 0.5941 **2.** (a) 0.3292 (b) 0.8683 (c) 0.2099 (d) 0.1317 **3.** (a) 0.1526 (b) 0.4812 (c) 0.5678 **4.** (a) 0.7738 (b) 3.125×10^{-7} (c) 0.9988 (d) 3×10^{-5} **5.** (a) 0.2787 (b) 0.4059 **6.** (a) 0.2610 (b) 0.9923 **7.** (a) 0.2786 (b) 0.7064 (c) 0.1061 **8.** (a) 0.1318 (b) 0.8484 (c) 0.0549 (d) 0.3296 **9.** (a) 0.2938 (b) 0.6531 (c) 0.0027 (d) 0.9726 (e) 12.86 **10.** (a) 0.0039 (b) 0.2734 (c) 0.6367 (d) 0.9648 **11.** (a) 0.3125 (b) 0.0156 (c) 0.3438 (d) 3 **12.** (a) 0.2785 (b) 0.3417 (c) 120 **13.** (a) 0.0331 (b) 0.5695 **14.** (a) 0.4305 (b) 0.9619 (c) \$720 (d) 0.2059 **15.** (a) i. 1.4 ii. 1 iii. 1.058 iv. 0.0795 v. 0.0047 (b) i. 3.04 ii. 3 iii. 1.373 iv. 0.2670 v. 0.1390 **16.** 38.23 **19.** (a) i. 0.1074 ii. 7.9×10^{-4} iii. 0.3758 (b) at least 6 **20.** (a) $\frac{4}{3}$ (b) $\frac{10}{9}$ (c) $\frac{1}{6}$ (d) $\frac{5}{288}$ **21.** (a) 20 (b) 3.4641 **22.** (a) 102.6 (b) 0.000254 **23.** (a) i. 6 ii. 2.4 (b) i. 6 ii. 3.6 **24.** 0.1797 **25.** 1.6, 1.472 **26.** (a) 0.1841 (b) \$11.93 **27.** (a) \$8 (b) \$160

28. (a) 0.0702 (c)



29. (b)



30. (b) 0.8035 (c) 39.3

EXERCISE 17.1

1. (i) 0.6915 (ii) 0.9671 (iii) 0.9474 (iv) 0.9965 (v) 0.9756 (vi) 0.0054 (vii) 0.0287 (viii) 0.0594 (ix) 0.0073 (x) 0.8289 (xi) 0.6443 (xii) 0.0823 **2.** (i) 0.0360 (ii) 0.3759 (iii) 0.0623 (iv) 0.0564 (v) 0.0111 (vi) 0.2902 (vii) 0.7614 (viii) 0.0343 (ix) 0.6014 (x) 0.1450 (xi) 0.9206 (xii) 0.2668 (xiii) 0.7020 (xiv) 0.9132 (xv) 0.5203 (xvi) 0.8160 (xvii) 0.9388 (xviii) 0.7258

EXERCISE 17.2

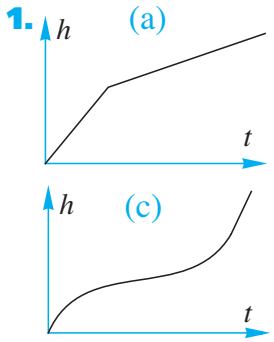
1. (a) 0.0228 (b) 0.9332 (c) 0.3085 (d) 0.8849 (e) 0.0668 (f) 0.9772 **2.** (a) 0.9772 (b) 0.0668 (c) 0.6915 (d) 0.1151 (e) 0.9332 (f) 0.0228 **3.** (a) 0.3413 (b) 0.1359 (c) 0.0489 **4.** (a) 0.6827 (b) 0.1359 (c) 0.3934 **5.** (a) 0.8413 (b) 0.4332 (c) 0.7734 **6.** (a) 0.1151 (b) 0.1039 (c) 0.1587 **7.** (a) 0.1587 (b) 0.6827 (c) 0.1359 **8.** (a) 0.1908 (b) 0.4754 (c) 16.88 **9.** (a) 0.1434 (b) 0.6595 **10.** (a) 0.2425 (b) 0.8413 (c) 0.5050 **11.** (a) –1.2816 (b) 0.2533 **12.** (a) 58.2243 (b) 41.7757 (c) 59.80 **13.** 39.11 **14.** 9.1660 **15.** 42% **16.** 0.7021 **17.** (i) 0.2903 (ii) 0.4583 (iii) 0.2514 **18.** 23% **19.** 0.5 **20.** 11% **21.** 5% **22.** 14% **23.** 1.8 **24.** 252 **25.** 0.1517 **26.** 0.3821 **27.** 0.22 **28.** 322 **29.** 0.1545 **30.** 7 **31.** 87 **32.** (a) i. 0.0062 ii. 0.0478 iii. 0.9460 (b) 0.0585 **33.** (a) \$5.11 (b) \$7.39 **34.** (a) 0.0062 (b) i. 0.7887 ii. 0.0324 (c) \$1472 **35.** (a) $\mu = 66.86, \sigma = 10.25$ (b) \$0.38*S* **36.** (a) $\mu = 37.2, \sigma = 28.2$ (b) 20 (19.9) **37.** (a) i. 0.3446 ii. 0.2347 (b) i. 0.3339 ii. 0.3852 (c) 0.9995

EXERCISE 18.1

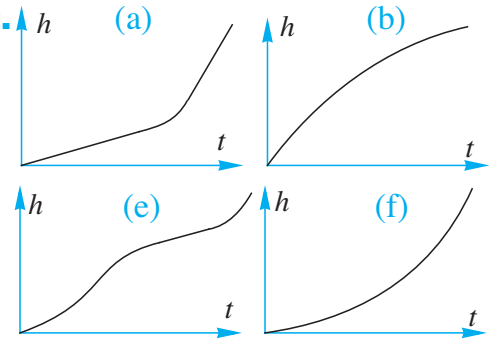
1. (a) $\frac{3}{4}$ (b) $\frac{3a}{4b}$ (c) –1 (d) 1 (e) $-\frac{15}{8}$ (f) 0 **2.** (a) 4 (b) 0.2 (c) 0.027 (d) 0.433 (e) –0.01 (f) –30 (g) 6.2 (h) 1 **3.** (a) 6 m/s (b) 30 m/s (c) $11 + 6h + h^2$ m/s **4.** 12 m/s **5.** $8 + 2h$ **6.** –3.49 °C/sec **7.** (a) 127π cm^3/cm (b) i. 19.6667π cm^3/cm ii. 1.9967π cm^3/cm iii. 0.2000π cm^3/cm **8.** 1.115 **9.** (a) –7.5 °C/min (b) $t = 2$ to $t = 6$ **10.** (a) 28 m (b) 14 m/s (c) average speed (d) 49 m (e) 49 m/s **11.** (a) \$1160, \$1345.6, \$1560.90, \$1810.64,\$ 2100.34 (b) \$220.07 per year

EXERCISE 18.2

1.



2.



EXERCISE 18.3

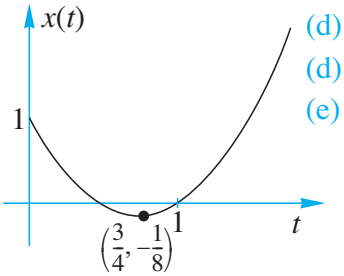
1. (a) $h + 2$ (b) $4 + h$ (c) $\frac{-1}{1+h}$ (d) $3 - 3h + h^2$ **2.** (a) 2 (b) 4 (c) –1 (d) 3 **3.** (a) $2a + h$

(b) $-(2a+h)$ (c) $(2a+2)+h$ (d) $3a^2+1+3ah+h^2$ (e) $-(3a^2+3ah+h^2)$

(f) $3a^2-2a+(3a-1)h+h^2$ (g) $\dfrac{-2}{a(a+h)}$ (h) $-\dfrac{1}{(a-1)(a-1+h)}$ (i) $\dfrac{1}{\sqrt{a+h}+\sqrt{a}}$

4. (a) 1 ; 1 (b) $2a+h$; $2a$ (c) $3a^2+3ah+h^2$; $3a^2$ (d) $4a^3+6a^2h+4ah^2+h^3$; $4a^3$

5. (a) (b) i. 3 ms^{-1} ii. 2 ms^{-1} iii. 1.2 ms^{-1}

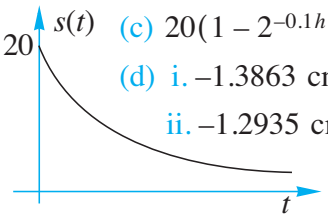


(d) $1+2h$

(d) Find (limit) as $h \to 0$

(e) $4t-3$

6. (a)



(b) i. 20 cm^2 ii. 17.41 cm^2

iii. 2.59 cm^2 iv. -1.29 cm^2day^{-1}

(c) $20(1-2^{-0.1h})$ cm^2day^{-1}

(d) i. -1.3863 cm^2day^{-1}

ii. -1.2935 cm^2day^{-1}

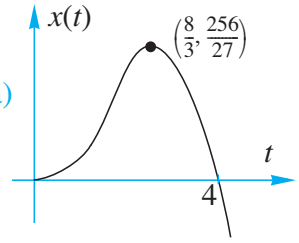
EXERCISE 18.4

1. (a) 3 (b) 8 (c) $-\dfrac{1}{9}$ (d) 1.39 (e) -1 (f) $\dfrac{17}{16}$ **2.** (a) 4.9 m (b) $4.9(h^2+2h)$ m (c) 9.8 m/s

3. (a) $8x$ (b) $10x$ (c) $12x^2$ (d) $15x^2$ (e) $16x^3$ (f) $20x^3$ **4.** (a) $4x$ (b) -1 (c) $-1+3x^2$ (d) $-x^{-2}$

(e) $-2(x+1)^{-2}$ (f) $0.5x^{-1/2}$ **5.** (a) 1 ms^{-1} (b) $(2-a)$ ms^{-1}

6. (a)



(b) i. 5 ms^{-1} ii. 4 ms^{-1} (c) $8t-3t^2$ ms^{-1} (d) $\dfrac{8}{3}$ sec

EXERCISE 19.1

1. (a) $5x^4$ (b) $9x^8$ (c) $25x^{24}$ (d) $27x^2$ (e) $-28x^6$ (f) $2x^7$ (g) $2x$ (h) $20x^3+2$ (i) $-15x^4+18x^2-1$

(j) $-\dfrac{4}{3}x^3+10$ (k) $9x^2-12x$ (l) $3+\dfrac{2}{5}x+4x^3$ **2.** (a) $-\dfrac{3}{x^4}$ (b) $\dfrac{3}{2}\sqrt{x}$ (c) $\dfrac{5}{2}\sqrt{x^3}$ (d) $\dfrac{1}{3\sqrt[3]{x^2}}$ (e) $\dfrac{2}{\sqrt{x}}$

(f) $9\sqrt{x}$ (g) $\dfrac{1}{\sqrt{x}}+\dfrac{3}{x^2}$ (h) $\dfrac{3}{2}\sqrt{x}-\dfrac{1}{2\sqrt{x^3}}$ (i) $\dfrac{10}{3\sqrt[3]{x}}-9$ (j) $5-\dfrac{1}{2\sqrt{x}}-\dfrac{8}{5x^3}$ (k) $\dfrac{4}{\sqrt{x}}-\dfrac{15}{x^6}+\dfrac{1}{2}$

(l) $-\dfrac{1}{2\sqrt{x^3}}-\dfrac{1}{\sqrt{x}}+x^2$ **3.** (a) $\dfrac{3}{2}\sqrt{x}+\dfrac{1}{\sqrt{x}}$ (b) $4x^3+3x^2-1$ (c) $3x^2+1$ (d) $\dfrac{1}{x^2}$ (e) $\dfrac{1}{\sqrt{x^3}}$

(f) $\dfrac{1}{2}-\dfrac{1}{4\sqrt{x^3}}$ (g) -7 (h) $2x-\dfrac{8}{x^3}$ (i) $2x-\dfrac{2}{x^2}-\dfrac{4}{x^5}$ (j) $\dfrac{1}{2}\sqrt{\dfrac{3}{x}}+\dfrac{1}{6\sqrt{x^3}}$ (k) $2x-\dfrac{12}{5}\sqrt[5]{x}+\dfrac{2}{5\sqrt[5]{x^3}}$

(l) $-\dfrac{3}{2\sqrt{x}}\left(\dfrac{1}{x}+1\right)\left(\dfrac{1}{\sqrt{x}}-\sqrt{x}\right)^2$

EXERCISE 19.2.1

1. $m_{PQ}=4+h$; $\lim\limits_{h\to 0} m_{PQ}=4$ **2.** $P(1,1), Q\left(1+h, \dfrac{2}{2+h}\right)$; $m_{PQ}=-\dfrac{1}{2+h}$; $\lim\limits_{h\to 0} m_{PQ}=-\dfrac{1}{2}$

3. -12 **4.** (a) 3 (b) $-\dfrac{1}{4}$ (c) 12 (d) 4 (e) 4 (f) $\dfrac{7}{6}$ (g) $-\dfrac{1}{12}$ (h) $\dfrac{53}{16}$ **5.** $\pm\sqrt{\dfrac{8}{3}}$ **6.** (a) $2x-12$ (b) -18

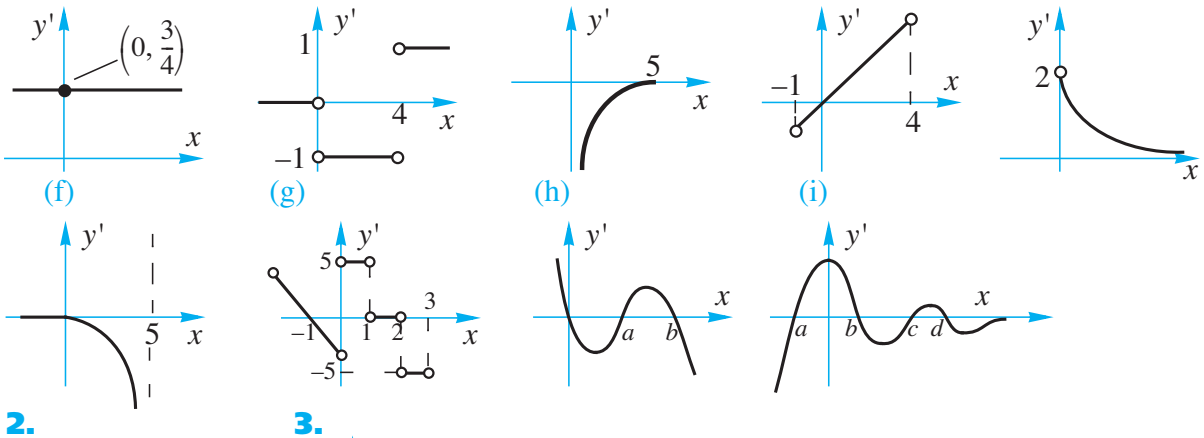
(c) (8, –32) **7.** (a) $-3x^2+3$ (b) 0 (c) $(\sqrt{2}, \sqrt{2}), (-\sqrt{2}, -\sqrt{2})$ **8.** (a) $\left(\pm\frac{\sqrt{2}}{2}, -\frac{1}{16}\right)$, (0,0)

(b) $\left\{x: \frac{-1}{\sqrt{2}} < x < 0\right\} \cup \left\{x: x > \frac{1}{\sqrt{2}}\right\}$ **9.** $x = \frac{1}{3}, -1$ **10.** (a) –2, 6, 3 (b) –2 **11.** $a = 1$ $b = -8$

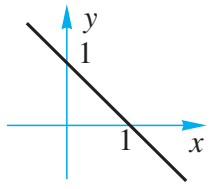
12. $f'(a+b) = 2(a+b) = 2a+2b$ **13.** (a) $4a^2 - 2a$, $a \geq 0$ (b) $4 - \frac{1}{a}$, $a > 0$ **14.** –56

EXERCISE 19.2.2

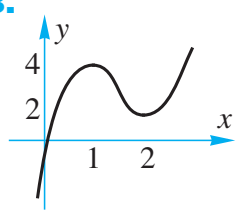
1. (a) (b) (c) (d) (e)



2.



3.



EXERCISE 19.2.3

1. (a) $48t^3 - \frac{1}{2\sqrt{t}}$ (b) $2n - \frac{2}{n^2} - \frac{4}{n^5}$ (c) $\frac{3}{2}\sqrt{r} + \frac{5}{6\sqrt[6]{r}} - \frac{1}{\sqrt{r}}$ (d) $2\theta - \frac{9}{2}\sqrt{\theta} + 3 - \frac{1}{2\sqrt{\theta}}$ (e) $40 - 3L^2$

(f) $-\frac{100}{v^3} - 1$ (g) $6l^2 + 5$ (h) $2\pi + 8h$ (i) $4n^3 - \frac{1}{3\sqrt[3]{n^2}} + \pi$

2. (a) $\frac{8}{3t^3}$ (b) $2\pi r - \frac{20}{r^2}$ (c) $\frac{5}{2}s^{3/2} + \frac{3}{s^2}$ (d) $-\frac{6}{t^4} + \frac{2}{t^3} - \frac{1}{t^2}$ (e) $-\frac{4}{b^2} + \frac{1}{2b^{3/2}}$ (f) $3m^2 - 4m - 4$

EXERCISE 19.3

1. (a) $3x^2 - 5x^4 + 2x + 2$ (b) $6x^5 + 10x^4 + 4x^3 - 3x^2 - 2x$ (c) $-\frac{4}{x^5}$ (d) $6x^5 + 8x^3 + 2x$

2. (a) $-\frac{2}{(x-1)^2}$ (b) $\frac{1}{(x+1)^2}$ (c) $\frac{1-x^2-2x}{(x^2+1)^2}$ (d) $\frac{-(x^4+3x^2+2x)}{(x^3-1)^2}$ (e) $\frac{2x^2+2x}{(2x+1)^2}$ (f) $\frac{1}{(1-2x)^2}$

3. (a) $(\sin x + \cos x)e^x$ (b) $\ln x + 1$ (c) $e^x(2x^3 + 6x^2 + 4x + 4)$ (d) $4x^3\cos x - x^4\sin x$

(e) $-\sin^2 x + \cos^2 x$ (f) $2x\tan x + (1+x^2)\sec^2 x$ (g) $\frac{4}{x^3}(x\cos x - 2\sin x)$

(h) $e^x(x\cos x + x\sin x + \sin x)$ (i) $(\ln x + 1 + x\ln x)e^x$

4. (a) $\frac{\sin x - x\cos x}{\sin^2 x}$ (b) $\frac{-[\sin x(x+1) + \cos x]}{(x+1)^2}$ (c) $\frac{e^x}{(e^x+1)^2}$ (d) $\frac{2x\cos x - \sin x}{2x\sqrt{x}}$ (e) $\frac{\ln x - 1}{(\ln x)^2}$

(f) $\dfrac{(x+1)-x\ln x}{x(x+1)^2}$ (g) $\dfrac{xe^x+1}{(x+1)^2}$ (h) $\dfrac{-2}{(\sin x-\cos x)^2}$ (i) $\dfrac{x^2-x+2x\ln x}{(x+\ln x)^2}$ **5.** (a) $-5e^{-5x}+1$

(b) $4\cos 4x+3\sin 6x$ (c) $-\dfrac{1}{3}e^{-\frac{1}{3}x}-\dfrac{1}{x}+18x$ (d) $25\cos 5x+6e^{2x}$ (e) $4\sec^2 4x+2e^{2x}$

(f) $-4\sin(4x)+3e^{-3x}$ (g) $\dfrac{4}{4x+1}-1$ (h) 0 (i) $\dfrac{1}{2}\cos\left(\dfrac{x}{2}\right)-2\sin 2x$ (j) $7\cos(7x-2)$ (k) $\dfrac{1}{2\sqrt{x}}-\dfrac{1}{x}$

(l) $\dfrac{1}{x}+6\sin 6x$ **6.** (a) $2x\cos x^2+2\sin x\cos x$ (b) $2\sec^2 2\theta-\dfrac{\cos\theta}{\sin^2\theta}$ (c) $\dfrac{1}{2\sqrt{x}}\cos\sqrt{x}$ (d) $\dfrac{1}{x^2}\sin\left(\dfrac{1}{x}\right)$

(e) $-3\sin\theta\cdot\cos^2\theta$ (f) $e^x\cos(e^x)$ (g) $\dfrac{1}{x}\sec^2(\log_e x)$ (h) $\dfrac{-\sin 2x}{\sqrt{\cos 2x}}$ (i) $-\cos\theta\cdot\sin(\sin\theta)$

(j) $4\sin\theta\cdot\sec^2\theta$ (k) $-5\cos 5x\cdot\csc^2(5x)$ (l) $-6\csc^2(2x)$

7. (a) $2e^{2x+1}$ (b) $-6e^{4-3x}$ (c) $-12xe^{4-3x^2}$ (d) $\dfrac{1}{2}\sqrt{e^x}$ (e) $\dfrac{1}{2\sqrt{x}}e^{\sqrt{x}}$ (f) e^{2x+4} (g) $2xe^{2x^2+4}$

(h) $-\dfrac{6}{e^{3x+1}}$ (i) $(6x-6)e^{3x^2-6x+1}$ (j) $\cos(\theta)e^{\sin\theta}$ (k) $2\sin(2\theta)e^{-\cos 2\theta}$ (l) $2x$ (m) $\dfrac{2e^{-x}}{(e^{-x}+1)^2}$

(n) $3(e^x+e^{-x})(e^x-e^{-x})^2$ (o) e^{x+2} (p) $(-2x+9)e^{-x^2+9x-2}$ **8.** (a) $\dfrac{2x}{x^2+1}$ (b) $\dfrac{\cos\theta+1}{\sin\theta+\theta}$

(c) $\dfrac{e^x+e^{-x}}{e^x-e^{-x}}$ (d) $-\dfrac{1}{x+1}$ (e) $\dfrac{3}{x}(\ln x)^2$ (f) $\dfrac{1}{2x\sqrt{\ln x}}$ (g) $\dfrac{1}{2(x-1)}$ (h) $\dfrac{-3x^2}{1-x^3}$ (i) $-\dfrac{1}{2(x+2)}$

(j) $\dfrac{-2\sin x\cos x}{\cos^2 x+1}$ (k) $\dfrac{1}{x}+\cot x$ (l) $\dfrac{1}{x}+\tan x$

9. (a) $\ln(x^3+2)+\dfrac{3x^3}{x^3+2}$ (b) $\dfrac{\sin^2 x}{2\sqrt{x}}+2\sqrt{x}\sin x\cos x$ (c) $-\dfrac{1}{\sqrt{\theta}}\sin\sqrt{\theta}\cdot\cos\sqrt{\theta}$

(d) $(3x^2-4x^4)e^{-2x^2+3}$ (e) $-(\ln x+1)\sin(x\ln x)$ (f) $\dfrac{1}{x\ln x}$

(g) $\dfrac{(2x-4)\sin(x^2)-2x\cos(x^2)(x^2-4x)}{(\sin x^2)^2}$ (h) $\dfrac{10[\ln(10x+1)-1]}{[\ln(10x+1)]^2}$ (i) $(\cos 2x-2\sin 2x)e^{x-1}$

(j) $2x\ln(\sin 4x)+4x^2\cot 4x$ (k) $(\cos\sqrt{x}-\sin\sqrt{x})\dfrac{1}{2\sqrt{x}}e^{-\sqrt{x}}$

(l) $-(2\sin x+2x\cos x)\cdot\sin(2x\sin x)$ (m) $\dfrac{e^{5x+2}(9-20x)}{(1-4x)^2}$ (n) $\dfrac{\cos^2\theta+\sin^2\theta\ln(\sin\theta)}{\sin\theta\cos^2\theta}$

(o) $\dfrac{x+2}{2(x+1)\sqrt{x+1}}$ (p) $\dfrac{2x^2+2}{\sqrt{x^2+2}}$ (q) $\dfrac{10x^3+9x^2+4x+3}{3(x+1)^{2/3}}$ (r) $\dfrac{3x^2(3x^3+1)}{2\sqrt{x^3+1}}$

(s) $\dfrac{2}{x^2+1}-\dfrac{1}{x^2}\ln(x^2+1)$ (t) $\dfrac{2}{x(x+2)}$ (u) $\dfrac{2-x}{2x^2\sqrt{x-1}}$ (v) $\dfrac{-x^2+x-9}{\sqrt{x^2+9}}\cdot e^{-x}$ (w) $\dfrac{7x^3-12x^2-8}{2\sqrt{2-x}}$

(x) $nx^{n-1}\ln(x^n-1)+\dfrac{nx^{2n-1}}{x^n-1}$ **10.** $x=1$ **11.** 0 **12.** 0 **13.** 1 **14.** $-2e$ **15.** (a) $\cos^2 x-\sin^2 x$

(b) $\dfrac{\pi}{180}\cos x°$ (c) $-\dfrac{\pi}{180}\sin x°$ **16.** (b) i. $2x\sin x\cos x+x^2\cos^2 x-x^2\sin^2 x$

ii. $e^{-x^3}(2\cos 2x\ln\cos x-3x^2\sin 2x\ln\cos x-\sin 2x\tan x)$ **17.** (a) i. $-\dfrac{3}{x}(\ln x)^2$ ii. $-\dfrac{3x^2}{1-x^3}$

(b) i. $-2e^{-2x}\cdot\cos(e^{-2x})$ ii. $-2x\cos x^2\cdot e^{-\sin x^2}$ **18.** $-\frac{1}{5}k$ **19.** $x = a, b, \frac{mb+na}{m+n}$

20. $\{\theta : n\tan\theta^m\cdot\tan\theta^n = m\theta^{m-n}\}$ **21.** (a) $-4\csc(4x)$ (b) $2\sec(2x)\tan(2x)$

(c) $3\cot(3x)\csc(3x)$ (d) $-3\sin(3x)$ (e) $\csc^2\left(\frac{\pi}{4}-x\right)$ (f) $-2\sec(2x)\tan(2x)$

22. (a) $2x\sec(x^2)\tan(x^2)$ (b) $\sec^2 x$ (c) $\tan x$ (d) $-3\cot^2 x\csc^2 x$ (e) $x\cos x+\sin x$

(f) $-2\cot x\csc^2 x$ (g) $4x^3\csc(4x)-4x^4\cot(4x)\csc(4x)$ (h) $2\cot x\sec^2(2x)-\csc^2 x\tan(2x)$

(i) $\frac{\sec x\tan x-\sin x}{2\sqrt{\cos x+\sec x}}$ **23.** (a) $e^{\sec x}\sec x\tan x$ (b) $e^x\sec(e^x)\tan(e^x)$

(c) $e^x\sec(x)+e^x\sec(x)\tan(x)$ (d) $\frac{-\csc^2(\log x)}{x}$ (e) $-5\csc(5x)\sec(5x)$

(f) $\frac{\cot(x)}{x}-\csc^2(x)\log x$ (g) $-\cos x\cot(\sin x)\csc(\sin x)$ (h) $-\cos(\csc x)\cot x\csc x$ (i) 0

EXERCISE 19.4

1. i. $20x^3$ ii. $48(1+2x)^2$ iii. $\frac{2}{x^3}$ iv. $\frac{2}{(1+x)^3}$ v. 2 vi. $\frac{6}{(x-2)^3}$ vii. $\frac{42}{x^8}$

viii. $24(1-2x)$ ix. $-\frac{1}{x^2}$ x. $\frac{-2(x^2+1)}{(1-x^2)^2}$ xi. $-16\sin 4\theta$ xii. $2\cos x-x\sin x$

xiii. $6x^2\cos x+6x\sin x-x^3\sin x$ xiv. $\frac{1}{x}$ xv. $\frac{10}{(2x+3)^3}$ xvi. $6xe^{2x}+12x^2e^{2x}+4x^3e^{2x}$

xvii. $\frac{8\sin 4x-15\cos 4x}{e^x}$ xviii. $2\cos x^2-4x^2\sin x^2$ xix. $\frac{-48(x^2+2x^5)}{(4x^3-1)^3}$ xx. $\frac{10}{(x-3)^3}$

2. $\frac{6\ln x-5}{x^4}, \frac{n^2\ln x+n\ln x-2n-1}{x^{n+2}}$ **3.** $f(x) = \left(\frac{x+1}{x-1}\right)^n \Rightarrow f''(x) = \frac{4n(n+x)}{(x^2-1)^2}\left(\frac{x+1}{x-1}\right)^n$

4. $2+\frac{1}{8\sqrt{2}}$ **5.** -1 **6.** $[0,1.0768[\cup]3.6436,2\pi]$

EXERCISE 20.1

1. (a) $y=7x-10$ (b) $y=-4x+4$ (c) $4y=x+5$ (d) $16y=-x+21$ (e) $4y=x+1$ (f) $4y=x+2$
(g) $y=28x-48$ (h) $y=4$ **2.** (a) $7y=-x+30$ (b) $4y=x-1$ (c) $y=-4x+14$ (d) $y=16x-79$
(e) $2y=9-8x$ (f) $y=-4x+9$ (g) $28y=-x+226$ (h) $x=2$ **3.** (a) $y=2ex-e$ (b) $y=e$ (c) $y=\pi$
(d) $y=-x$ (e) $y=x$ (f) $ey=(2e-1)x-e^2+2e-1$ (g) $y=ex$ (h) $y=2x+1$

4. (a) $2ey=-x+2e^2+1$ (b) $x=1$ (c) $x=\pi$ (d) $y=x-2\pi$ (e) $y=-x+\pi$
(f) $(2e-1)y=-ex+3e^2-4e+1$ (g) $ey=-x$ (h) $2y=-x+2$ **5.** A: $y=28x-44$,
B: $y=-28x-44$, Isosceles. $z\equiv(0,a^2-3a^4)$ **6.** 2 sq. units, $y=2$ $x=1$ **7.** $4y=3x$

8. $by=\sqrt{a^2-b^2}x$ **9.** $y=4x-9$ **10.** $y=\log_e 4$ **11.** $8y=4(\pi+2)x-\pi^2$;

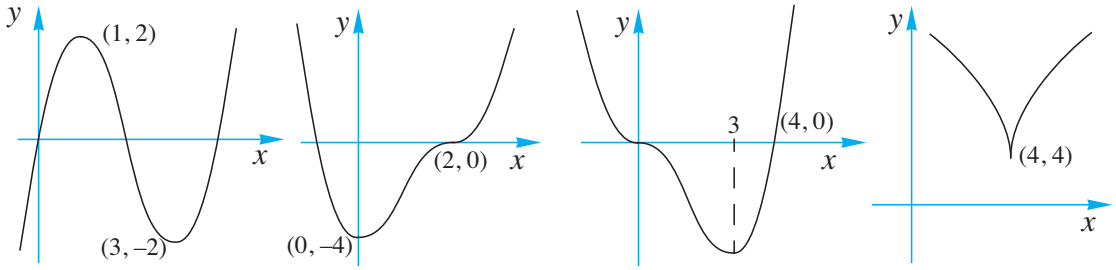
$4(\pi+2)y=-8x+4\pi+\pi^2$ **12.** A: $y=-8x+32$, B: $y=6x+25$, $\left(\frac{1}{2},28\right)$ **13.** $y=-x$, Tangents:

$y=\frac{1}{2}, y=-\frac{1}{2}$ $\left(-\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{2},-\frac{1}{2}\right)$ tangent and normal meet at $(0.5,-0.5)$

14. (a) $y=3x-7$ (b) $Q\equiv(2,-1)$ **15.** $m=-2, n=5$

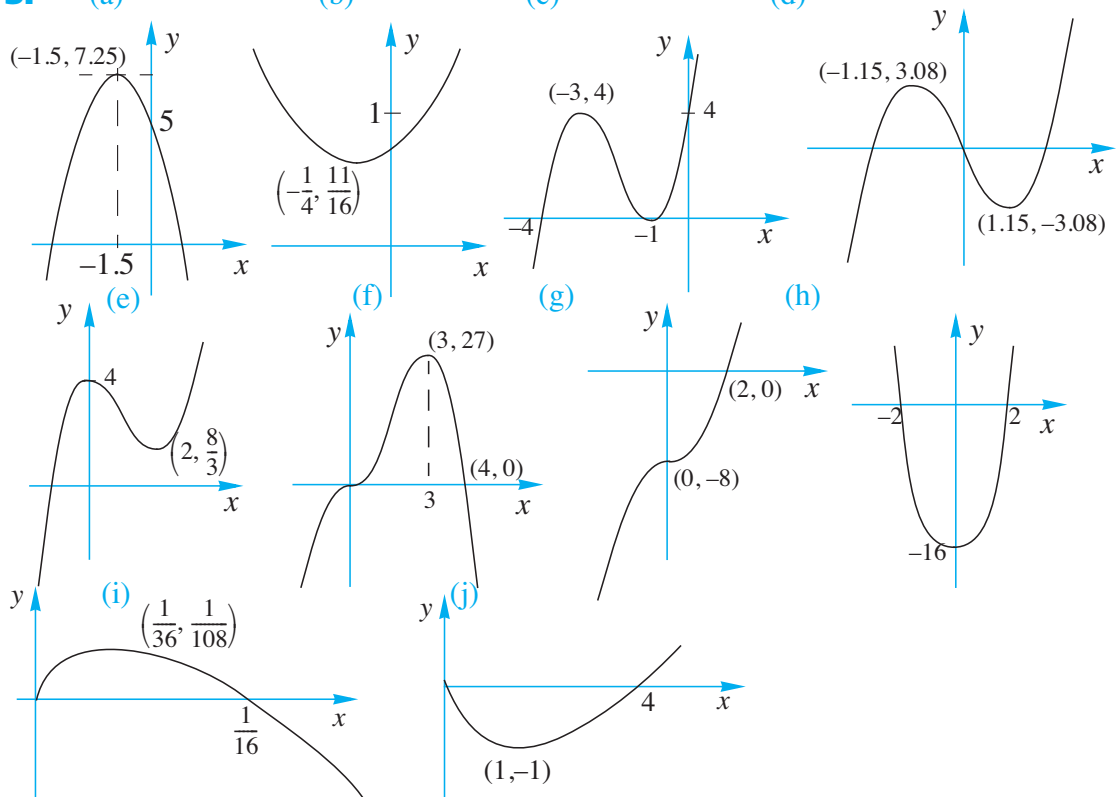
EXERCISE 20.2

1. (a) (b) (c) (d)

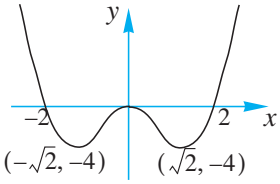


2. (a) max at (1, 4) (b) min at $\left(-\frac{9}{2}, -\frac{81}{4}\right)$ (c) min at (3, –45) max (–3, 63) (d) max at (0, 8), min at (4, -24) (e) max at (1, 8), min at (–3, –24) (f) min at $\left(\frac{1+\sqrt{13}}{3}, \frac{70-26\sqrt{13}}{27}\right)$, max at $\left(\frac{1-\sqrt{13}}{3}, \frac{70+26\sqrt{13}}{27}\right)$ (g) min at (1, –1) (h) max at (0, 16), min at (2, 0), min at (–2, 0) (i) min at (1, 0) max at $\left(-\frac{1}{3}, \frac{32}{27}\right)$ (j) min at $\left(\frac{4}{9}, -\frac{4}{27}\right)$ (k) min at (2, 4), max at (–2, –4) (l) min at (1, 2), min at (–1, 2)

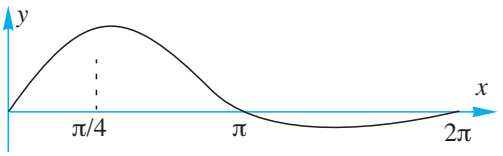
3. (a) (b) (c) (d)



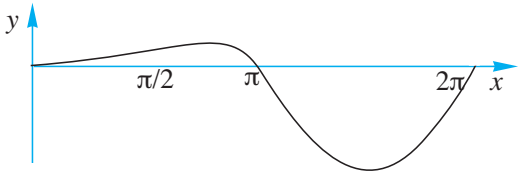
4. min at (1, –3), max at (–3, 29), non-stationary infl (–1, 13) **5.**



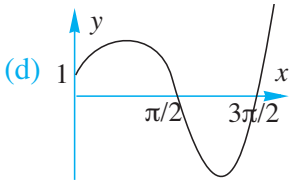
6. (a) i. $(\cos x - \sin x)e^{-x}$ ii. $-2\cos x.e^{-x}$ (b) i. $\frac{\pi}{4}, \frac{5\pi}{4}$ ii. $\frac{\pi}{2}, \frac{3\pi}{2}$ (c) Inf. $\left(\frac{\pi}{2}, e^{-\frac{\pi}{2}}\right)$ $\left(\frac{3\pi}{2}, -e^{-\frac{3\pi}{2}}\right)$

(d) 

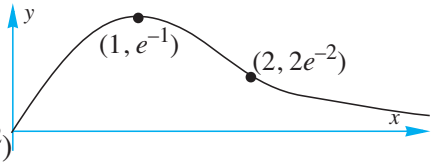
7. (a) i. $e^x(\sin x + \cos x)$ ii. $2e^x \cos x$ (b) i. $x = \frac{3\pi}{4}, \frac{7\pi}{4}$ ii. $x = \frac{\pi}{2}, \frac{3\pi}{2}$ (c) st.pts. $\left(\frac{3\pi}{4}, \frac{1}{\sqrt{2}}e^{\frac{3\pi}{4}}\right), \left(\frac{7\pi}{4}, -\frac{1}{\sqrt{2}}e^{\frac{7\pi}{4}}\right)$ Infl. pts. $\left(\frac{\pi}{2}, e^{\frac{\pi}{2}}\right), \left(\frac{3\pi}{2}, -e^{\frac{3\pi}{2}}\right)$

(d) 

8. (a) i. $e^x(\cos x - \sin x)$ ii. $-2\sin x.e^x$ (b) i. $\frac{\pi}{4}, \frac{5\pi}{4}$ ii. $0, \pi, 2\pi$ (c) st.pts. $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}e^{\frac{\pi}{4}}\right), \left(\frac{5\pi}{4}, -\frac{1}{\sqrt{2}}e^{\frac{5\pi}{4}}\right)$ Inf. pts. $(0,1), (\pi, -e^{\pi}), (2\pi, e^{2\pi})$

(d) 

9. (a) i. $(1-x)e^{-x}$ ii. $(x-2)e^{-x}$
(b) i. $x = 1$ ii. $x = 2$
(c) st. pt. $(1, e^{-1})$ Inf. pt. $(2, 2e^{-2})$

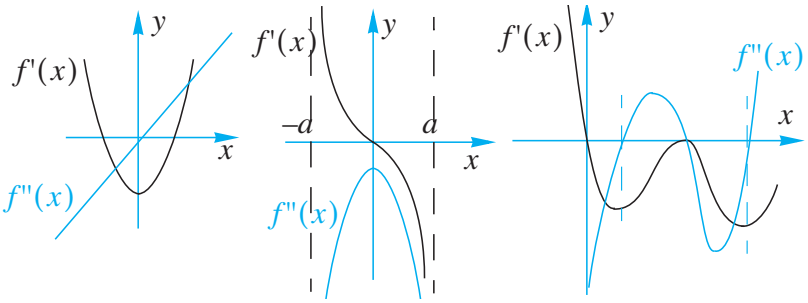


10. (a) 8 (b) 0 (c) 4 (d) $27\sqrt[3]{9} \approx 56.16$ **11.** i. min value –82. ii. max value 26

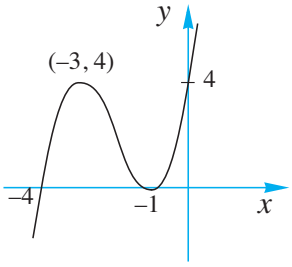
12. (a) A i. Yes ii. non-stationary pt of inflect. B. i. Yes ii. Stationary point (local/global min) C. i. Yes ii. non-stationary pt of inflect.
(b) A. i. No ii. Local/global max B. i. No. ii. Local/global min C. i. Yes ii. Stationary point (local max)
(c) A. i. Yes ii. Stationary point (local/global max) B. i. Yes ii. Stationary point (local min) C. i. Yes ii. non-stationary pt of inflect.
(d) A. i. Yes ii. Stationary pt (local/global max) B. i. No ii. Local min C. i. Yes ii. Stationary point (local max)
(e) A. i. No ii. Cusp (local min) B. i. Yes ii. Stationary pt of inflect C. i. Yes ii. Stationary point (local max)
(f) A. i. Yes ii. Stationary point (local/global max) B. i. Yes ii. Stationary point (local/global min) C. i. No ii. Tangent parallel to y–axis.

13. (a) i. A ii. B iii. C (b) i. C ii. B iii. A

14. i. ii. iii.

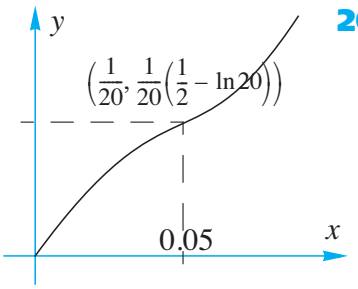


15. $y = x^3 + 6x^2 + 9x + 4$



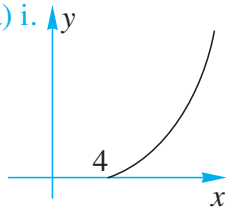
16. $f(x) = \frac{1}{3}x^3 - x^2 - 3x - 6$ **17.** $f(x) = 3x^5 - 20x^3$

18.

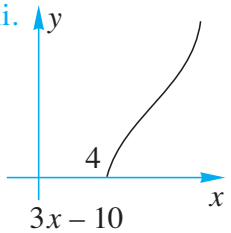


19. $m = -0.5, n = 1.5$

20. (a) i.



ii.



(b) i. $\frac{3}{2}\sqrt{x-4}$ ii. $\frac{3x-10}{2\sqrt{x-4}}$

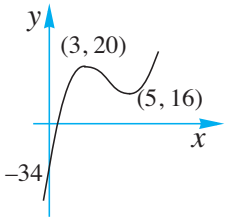
21. $a = 2, b = -3, c = 0$ **22.** Stationary points: local min at $(-1, 0)$ and local max at $(1, 4e^{-1})$. inflection. pts are: $(1+\sqrt{2}, (6+4\sqrt{2})e^{-(1+\sqrt{2})})$ and $(1-\sqrt{2}, (6-4\sqrt{2})e^{-(1-\sqrt{2})})$

23. Absolute min at $\sim \left(\frac{-3+\sqrt{13}}{2}, -2.1733\right)$, Local max at $\sim \left(\frac{-3-\sqrt{13}}{2}, 0.2062\right)$

Inflection pts at $\sim (-0.4384, -1.4489)$ and $(-4.5615, 0.1488)$

24. – 27. are left as questions for classroom discussion.

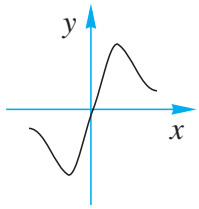
28. $a = 1, b = -12,$
$c = 45, d = -34$



29. (b) $b = 1$

(c) $a = \frac{1}{\sqrt{2}}$

(d) $f(x) = \frac{1}{\sqrt{2}} x e^{-x^2}$



30. (a) 2.7983, 6.1212, 9.3179 (b) Make use of Graphics calculator to verify your sketch.

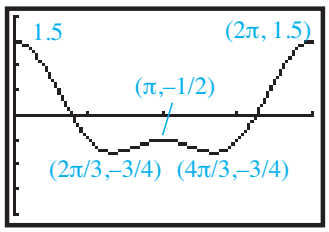
EXERCISE 20.3

1. (a) L.min at $x = \frac{4}{\sqrt{3}}$. L.max at $x = -\frac{4}{\sqrt{3}}$ (b) L.max at $x = 0$, L.min at $x = \pm 1$

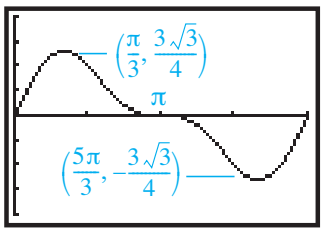
(c) L.max at $x = 0.25$ (d) L.max at $x = 1$ (e) none (f) L.max at $x = 0.5$, L.min at $x = 1$ and $x = 0$

(g) L.max at $x = 1$, L.min at $x = -1$ (h) none **2.** (a) max = 120, min $= -\frac{128}{3\sqrt{3}}$ (b) max = 224, min = –1 (c) max = 0.5, min = 0 (d) max = 1, min = 0.

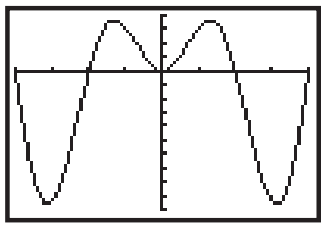
3.



4.

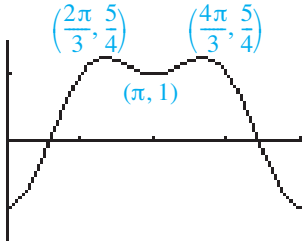


5. stationary points occur where tan$x = x$



6. (a) L.min at (1, 2); Infl. pt at $\left(3, \sqrt{3} + \frac{1}{3}\sqrt{3}\right)$ (b) L.min at (1, 2); L.max at (–3, –6) (c) none

7.



For **Q.8. – Q.11** verify your graphs with graphics calculator

8. (a) Global. min at $(0, 0)$; local max at $(2, 4e^{-2})$

Infl. pts $(2-\sqrt{2}, (6-4\sqrt{2})e^{-(2-\sqrt{2})})$

$(2+\sqrt{2}, (6+4\sqrt{2})e^{-(2+\sqrt{2})})$

(b) Global. max: $(0, e^4)$. Infl. pt: $\left(\pm\frac{1}{\sqrt{2}}, e^{3.5}\right)$ (c) L.max: $\left(-2, -\frac{1}{2}e\right)$

9. (a) Global max at (e, e^{-1}). Infl. pt at $(e^{1.5}, 1.5e^{-1.5})$ (b) Global min at $\left(\frac{1}{\sqrt{2}}, 2+\frac{1}{2}\ln 2\right)$

(c) Global min at (2, 1 + ln2); Infl. pt at (4, 2 + ln4) (d) none

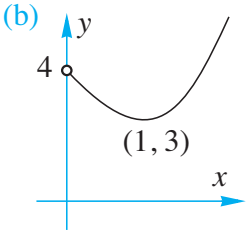
10. (a) $f'(x) = (x-2)^{a-1}(x+2)^{b-1}((a+b)x+2(a-b))$

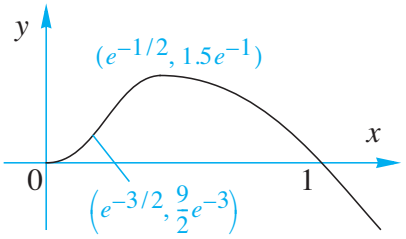
(b) i. $f(x) = \frac{x-2}{x+2}$; none ii. $f(x) = (x-2)^2(x+2)$ local max at $\left(-\frac{2}{3}, \frac{256}{27}\right)$; local min at (2, 0)

iii. $f(x) = (x-2)^2(x+2)^2$; local min at (±2, 0), local max at (0, 16).

11. (a) Global. min at $(1, c-1)$ **12.**

(b)





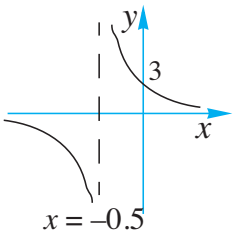
13. Global max at $(e^{0.5}, 0.5e^{-1})$; Infl. pt at $(e^{5/6}, (5/6)e^{-5/3})$.

EXERCISE 20.4

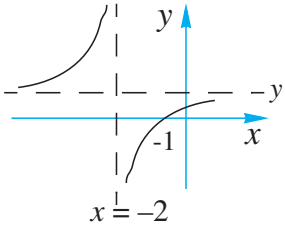
1. (a) $y = 2, x = -1$ (b) $y = 1, x = -\frac{1}{3}$ (c) $y = \frac{1}{2}, x = -\frac{1}{4}$ (d) $y = -1, x = -3$ (e) $y = 3, x = 0$

(f) $y = 5, x = 2$

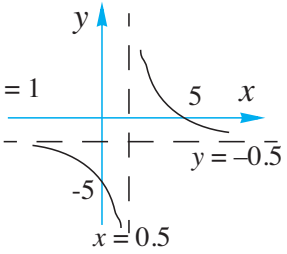
3. (a)



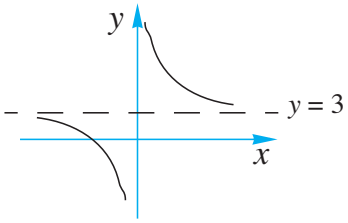
(b)



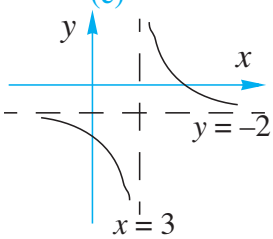
(c)



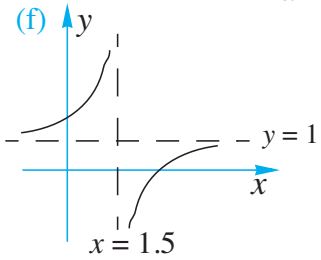
(d)



(e)

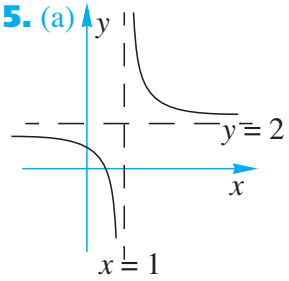


(f)

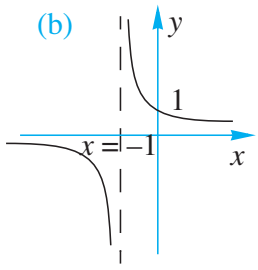


4. $a = 2, c = 4$

5. (a)

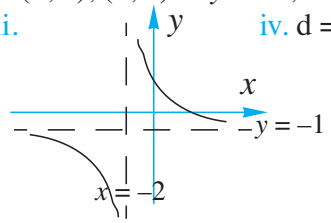


(b)



6. (a) i. (0, 1), (2, 0) ii. $y = -1, x = -2$

iii.

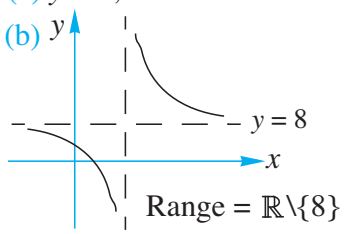


iv. d $= \mathbb{R}\backslash\{-2\}$

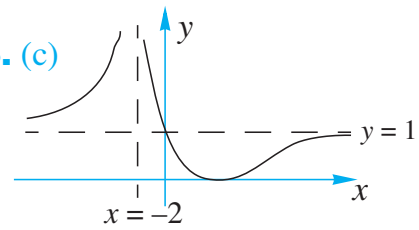
(b) $f^{-1}: \mathbb{R}\backslash\{-1\} \mapsto \mathbb{R}$, where $f^{-1}(x) = \frac{2(1-x)}{(1+x)}$

7. (a) $y = 8, x = 3$

(b)



6. (c)



EXERCISE 21.1

1.(a) i. $x < 0$ ii. $x > 4$ iii. $0 \le x \le 4$ (b) i. $-1 < x < 2$ ii. $x < -1, 2 < x < 5$ iii. $\varnothing$
(c) i. $-1 < x < 1$ ii. $x < -1$ iii. $x \ge 1$ (d) i. $0 < x < 1$ ii. $2 < x < 3$ iii. $x < 0, 1 \le x < 2$
(e) i. $\varnothing$ ii. $-2 < x < 4$ iii. $\varnothing$ (f) i. $-4 < x < -1, 2 < x < 5$ ii. $-1 < x < 2, 5 < x < 8$ iii. $\varnothing$

EXERCISE 21.2

1. 4.4 (4 deer per year, to nearest integer) **2.** (a) 200 cm^3 (b) 73.5 $cm^3 day^{-1}$ **3.** (a) 75 (b) No
4. (a) \$207.66 (b) \$ 40.79 per year (c) \$41.54 per year **5.** i. 2.50 ii. 3.33 iii. 2.50
6. (a) $1230 < x < 48770$ (approx) (b) i. $0 \le x < 25000$ ii. $25000 < x \le 50000$ **7.** 66667 (to nearest integer), 1446992 (to nearest integer) **8.** (b) 133.33 (d) 46.67 (e) $0 < x < 5700$

9. (a) $D'(x) = \dfrac{-40000(2x+12)}{(x^2+12x+20)^2}$ $5 \le x \le 18$ (b) 22.22 (22 items/dollar) **10.** (a) $\dfrac{3000}{(x+32)^2}$

(b) i. $x \ge 0$ ii. $x \in \varnothing$ **11.** (a) i. 0 mm/s ii. ~ 90.69 mm/s (b) 0.6 sec **12.** (a) 8.53 cm/s (b) (c) never
13. $-e^{-1}$ ms^{-2}

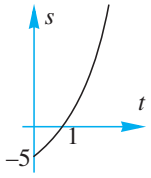
EXERCISE 21.3

1. (a) i. $v = -\dfrac{1}{(t-1)^2}, t > 1$ ii. $a = \dfrac{2}{(t-1)^3}, t > 1$ (b) i. $v = 2(e^{2t} - e^{-2t}), t \ge 0$

ii. $a = 4(e^{2t} + e^{-2t}), t \ge 0$ (c) i. $v = a - 2bte^{-t^2}, t \ge 0$ ii. $a = 2be^{-t^2}(2t^2 - 1), t \ge 0$

2. (a) 8 ms^{-1} (b) never at rest (c) i. 5m from O in negative direction ii. 4 ms^{-1} (d) 40 m

(e)



3. (a) 1 ms^{-1} (b) never (c) $t = \dfrac{1}{3}$ or $t = 1$ (d) 20 ms^{-2}

4. (a) $v = -6t^2 + 12$; $a = -12t$ (b) ~ 1.41 sec (c) once (d) use graphics calculator

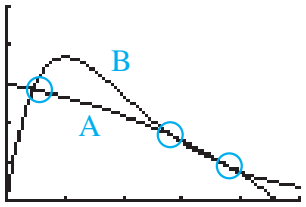
5. (a) 3 m in positive direction (b) i. 5 m ii. 2 m (c) 5 ms^{-1} (e) oscillation about origin with amplitude 5 m and period 2π sec **7.** (a) 100 m, in negative direction (b) 3 times (c) i. 80 ms^{-1}

ii. –34 ms^{-2} (d) 14.81m **8.** (a) max = 5 units, min = –1 unit (b) $\dfrac{\pi}{2}$ sec (c) i. $a = -12\cos(2t - \pi)$

ii. $a = -4(x-2)$ **9.** (a) 0.318 m above (b) i. $v = 3.75e^{-0.25t} - 3$ ii. $a = -0.9375e^{-0.25t}$

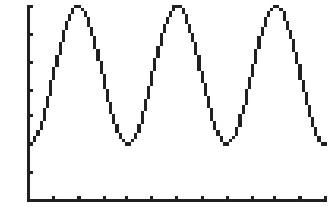
(c) 0.322 m (d) $a = -0.25(v+3)$ **10.** (a) $0 < t < 0.5$ or $t > 1$ (b) $t > 0.5$ (c) $t = 1$ or $1.68 \le t \le 5$.

11. (a) This question is best done using a graphics calculator:
(b) From the graph the particles pass each other three times.
(c) 0.45 sec; 2.85 sec; 3.87 sec
(d) i. $v_A = -0.3e^{0.3t}$ ms^{-1} ii. $v_B = 10e^{-t}(1-t)$ ms^{-1}
(e) Yes. On two occassions



12. (a) 2m in positive direction (b) i. 2 sec ii. never (c) 0.026 ms^{-2}

13. (a) (b) 0.295



window: $[0, 3\pi]$ by $[0, 14]$

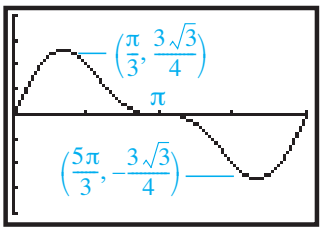
EXERCISES 21.4

1. 22.6 m **2.** (a) 1.5 kmh^{-1} (b) \$19.55 per km **3.** (a) 400 (b) \$46,400,000 **4.** \$273.86

5. \$0.40 **6.** 1.97 m **7.** 0.45 m^3 **8.** 5m by 5m **9.** 128

10. $r = \dfrac{50}{4+\pi} \approx 7.00$, dim of rect. $\dfrac{50}{4+\pi} \times \dfrac{50}{4+\pi}$ i.e., aprrox 7.00 m by 7.00 m **11.** $\theta = \dfrac{\pi}{6}$

12. (a)

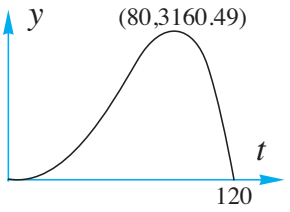


(b) $\dfrac{3\sqrt{3}}{2}$ units (c) At points of inflections, when $\cos x = -\dfrac{1}{4}$.

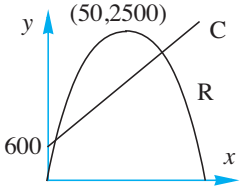
13. 648 m^2 **14.** (a) 10.5 (b) 5.25 **15.** 72 **16.** (a) $y = 100 - 2x$ (b) $A = x(100 - 2x), 0 < x < 50$

(c) $x = 25, y = 50$ **17.** (a) $\dfrac{100}{x} - \dfrac{1}{2}x, 0 < x < 10\sqrt{2}$ (b) $\dfrac{2000}{9}\sqrt{6} \approx 544.3$ cm^3

18. (a) 400 $ml\ s^{-1}$ (b) 40 sec (c)



19. (a)



(b) 8.38, 71.62 (c) $9 \le x \le 71$ (d) $80x - x^2 - 600$, \$1000 **20.** $\left(\sqrt{\dfrac{11}{2}}, \dfrac{7}{2}\right)$ & $\left(-\sqrt{\dfrac{11}{2}}, \dfrac{7}{2}\right)$

21. $5\sqrt{2}$ by $\dfrac{5}{2}\sqrt{2}$ **22.** 4 by $\dfrac{8}{3}$ **23.** $348 - 8\sqrt{170} \sim 243.7$ cm^2 **24.** 2 **25.** radius = $\sqrt{\dfrac{10}{3}}$ cm,

height = $2\sqrt{\dfrac{10}{3}}$ cm **26.** $\sqrt[3]{\dfrac{15}{\pi}}$ **27.** 5 cm **28.** (a) $h = \dfrac{24r^2}{r^2 - 144}$ (b) $\dfrac{8\pi r^4}{r^2 - 144}$ (c) $r = 12\sqrt{2}$,

$h = 48$ **29.** $r : h = 1 : 2$ **30.** $\sim (0.55, 1.31)$ **31.** (b) 2.5 m **32.** altitude = $\dfrac{1}{3}$ height of cone

33. ~ 1.640 m wide and 1.040 m high **34.** $\dfrac{2\sqrt{2}}{\sqrt{3}}\pi$ **35.** where XP : PY = $b : a$ **36.** 5 km

37. $r : h = 1 : 1$ **38.** $\dfrac{4}{3}$ cm **39.** 2 : 1 **40.** $\dfrac{10}{\sqrt{3\pi}}$ **41.** 0.873 km from P

42. (b) $r = 3\sqrt{2}$, $h = 6\sqrt{2}$ **43.** (b) when $\theta = \arcsin\left(\dfrac{5}{6}\right)$, i.e., approx. 6.030 km from P.

44. (a) $\tan\theta = \dfrac{xl}{x^2 + k(l+k)}$ (b) $x = \sqrt{k^2 + kl}$

45. (c) if $k < c$, swimmer should row directly to Q. **46.** (a) i. $\pi r^2 h + \dfrac{2}{3}\pi r^3$ ii. $3\pi r^2 + 2\pi rh$

(c) $r : h = 1:1$ **47.** $(a^{2/3} + b^{2/3})^{3/2}$ **48.** (b) 4 km along the beach (c) row directly to destination

EXERCISE 22.1

1. (a) $\dfrac{1}{4}x^4 + c$ (b) $\dfrac{1}{8}x^8 + c$ (c) $\dfrac{1}{6}x^6 + c$ (d) $\dfrac{1}{9}x^9 + c$ (e) $\dfrac{4}{3}x^3 + c$ (f) $\dfrac{7}{6}x^6 + c$ (g) $x^9 + c$

(h) $\dfrac{1}{8}x^4 + c$ **2.** (a) $5x + c$ (b) $3x + c$ (c) $10x + c$ (d) $\dfrac{2}{3}x + c$ (e) $-4x + c$ (f) $-6x + c$

(g) $-\dfrac{3}{2}x + c$ (h) $-x + c$ **3.** (a) $x - \dfrac{1}{2}x^2 + c$ (b) $2x + \dfrac{1}{3}x^3 + c$ (c) $\dfrac{1}{4}x^4 - 9x + c$

(d) $\dfrac{2}{5}x + \dfrac{1}{9}x^3 + c$ (e) $\dfrac{1}{3}x^{3/2} + \dfrac{1}{x} + c$ (f) $x^{5/2} + 4x^2 + c$ (g) $\dfrac{1}{3}x^3 + x^2 + c$ (h) $x^3 - x^2 + c$

(i) $x - \dfrac{1}{3}x^3 + c$ **4.** (a) $\dfrac{1}{3}x^3 - \dfrac{1}{2}x^2 - 6x + c$ (b) $\dfrac{1}{4}x^4 - \dfrac{2}{3}x^3 - \dfrac{3}{2}x^2 + c$ (c) $\dfrac{1}{4}(x-3)^4 + c$

(d) $\dfrac{2}{5}x^5 + \dfrac{1}{2}x^4 + \dfrac{1}{3}x^3 + \dfrac{1}{2}x^2 + c$ (e) $x + \dfrac{1}{2}x^2 - \dfrac{2}{3}x^{3/2} - \dfrac{2}{5}x^{5/2} + c$

(f) $\dfrac{2}{7}x^{7/2} + \dfrac{4}{5}x^{5/2} + \dfrac{2}{3}x^{3/2} - 2x + c$ **5.** (a) $\dfrac{1}{2}x^2 - 3x + c$ (b) $2u^2 + 5u + \dfrac{1}{u} + c$

(c) $-\dfrac{1}{x} - \dfrac{2}{x^2} - \dfrac{4}{3x^3} + c$ (d) $\dfrac{1}{2}x^2 + 3x + c$ (e) $\dfrac{1}{2}x^2 - 4x + c$ (f) $\dfrac{1}{3}t^3 + 2t - \dfrac{1}{t} + c$

6. (a) $\dfrac{4}{7}\sqrt[4]{x^7} + 2\sqrt{x} - 5x + c$ (b) $\dfrac{1}{3}x^3 + \dfrac{1}{2}x^2 - \dfrac{4}{7}x^{7/2} - \dfrac{4}{5}x^{5/2} + c$ (c) $-\dfrac{1}{2z^2} + \dfrac{2}{z} + 2z^2 + z + c$

(d) $\dfrac{1}{2}t^4 + t + c$ (e) $\dfrac{2}{5}\sqrt{t^5} - 2\sqrt{t^3} + c$ (f) $\dfrac{1}{3}u^3 + 2u^2 + 4u + c$ **8.** (a) $\dfrac{1}{8}(2x+3)^4 + c$

(b) $3\sqrt{x^2+4} + c$

EXERCISE 22.2

1. (a) $x^2 + x + 3$ (b) $2x - \dfrac{1}{3}x^3 + 1$ (c) $\dfrac{8}{3}\sqrt{x^3} - \dfrac{1}{2}x^2 - \dfrac{40}{3}$ (d) $\dfrac{1}{2}x^2 + \dfrac{1}{x} + 2x - \dfrac{3}{2}$ (e) $(x+2)^3$

(f) $\dfrac{3}{4}\sqrt[3]{x^4} + \dfrac{1}{4}x^4 + x$ (g) $\dfrac{1}{3}x^3 + 1$ (h) $x^4 - x^3 + 2x + 3$ **2.** $\dfrac{1}{2}x^2 + \dfrac{1}{x} + \dfrac{5}{2}$ **3.** \$3835.03

4. 9.5 **5.** $\dfrac{251\pi}{3}$ cm^3 **6.** 292 **7.** $\dfrac{5}{7}\sqrt{x^3} + \dfrac{23}{7}$ **8.** $(1, -8)$ **9.** $P(x) = 25 - 5x + \dfrac{1}{3}x^2$

10. $N = \dfrac{20000}{201}t^{2.01} + 500, t \geq 0$ **11.** (a) $y = -\dfrac{2}{5}x^2 + 4x$ (b) $y = \dfrac{1}{6}x^3 + \dfrac{5}{4}x^2 + 2x$

12. $y = 2(x^3 + x^2 + x)$ **13.** $f(x) = -\dfrac{3}{10}x^3 + \dfrac{49}{10}x - \dfrac{13}{5}$ **14.** Vol ~ 43202 cm^3 **15.** 110 cm^2

EXERCISE 22.3

1. (a) $\frac{1}{5}e^{5x}+c$ (b) $\frac{1}{3}e^{3x}+c$ (c) $\frac{1}{2}e^{2x}+c$ (d) $10e^{0.1x}+c$ (e) $-\frac{1}{4}e^{-4x}+c$ (f) $-e^{-x}+c$

(g) $-0.2e^{-0.5x}+c$ (h) $-2e^{1-x}+c$ (i) $5e^{x+1}+c$ (j) $e^{2-2x}+c$ (k) $3e^{x/3}+c$ (l) $2\sqrt{e^x}+c$

2. (a) $4\log_e x+c,\ x>0$ (b) $-3\log_e x+c,\ x>0$ (c) $\frac{2}{5}\log_e x+c,\ x>0$ (d) $\log_e(x+1)+c,\ x>-1$

(e) $\frac{1}{2}\log_e x+c,\ x>0$ (f) $x-2\log_e x-\frac{1}{x}+c,\ x>0$ (g) $\frac{1}{2}x^2-2x+\log_e x+c,\ x>0$

(h) $3\ln(x+2)+c$ **3.** (a) $-\frac{1}{3}\cos(3x)+c$ (b) $\frac{1}{2}\sin(2x)+c$ (c) $\frac{1}{5}\tan(5x)+c$ (d) $\cos(x)+c$

4. (a) $-\frac{1}{2}\cos(2x)+\frac{1}{2}x^2+c$ (b) $2x^3-\frac{1}{4}\sin(4x)+c$ (c) $\frac{1}{5}e^{5x}+c$ (d) $-\frac{4}{3}e^{-3x}-2\cos\left(\frac{1}{2}x\right)+c$

(e) $3\sin\left(\frac{x}{3}\right)+\frac{1}{3}\cos(3x)+c$ (f) $\frac{1}{2}e^{2x}+4\log_e x-x+c,\ x>0$ (g) $\frac{1}{2}e^{2x}+2e^x+x+c$

(h) $\frac{5}{4}\cos(4x)+x-\log_e x+c,\ x>0$ (i) $\frac{1}{3}\tan(3x)-2\log_e x+2e^{x/2}+c,\ x>0$

(j) $\frac{1}{2}e^{2x}-2x-\frac{1}{2}e^{-2x}+c$ (k) $\frac{1}{2}e^{2x+3}+c$ (l) $-\frac{1}{2}\cos(2x+\pi)+c$ (m) $\sin(x-\pi)+c$

(n) $-4\cos\left(\frac{1}{4}x+\frac{\pi}{2}\right)+c$ (o) $2e^{x/2}+4e^{-x/2}+c$ **5.** (a) $\frac{1}{16}(4x-1)^4+c$ (b) $\frac{1}{21}(3x+5)^7+c$

(c) $-\frac{1}{5}(2-x)^5+c$ (d) $\frac{1}{12}(2x+3)^6+c$ (e) $-\frac{1}{27}(7-3x)^9+c$ (f) $\frac{1}{5}\left(\frac{1}{2}x-2\right)^{10}+c$

(g) $-\frac{1}{25}(5x+2)^{-5}+c$ (h) $\frac{1}{4}(9-4x)^{-1}+c$ (i) $-\frac{1}{2}(x+3)^{-2}+c$ (j) $\ln(x+1)+c,\ x>-1$

(k) $\ln(2x+1)+c,\ x>-\frac{1}{2}$ (l) $-2\ln(3-2x)+c,\ x<\frac{3}{2}$ (m) $3\ln(5-x)+c,\ x<5$

(n) $-\frac{3}{2}\ln(3-6x)+c,\ x<\frac{1}{2}$ (o) $\frac{5}{3}\ln(3x+2)+c,\ x>-\frac{2}{3}$ **6.** (a) $-\frac{1}{2}\cos(2x-3)-x^2+c$

(b) $6\sin\left(2+\frac{1}{2}x\right)+5x+c$ (c) $\frac{3}{2}\sin\left(\frac{1}{3}x-2\right)-\ln(2x+1)+c$ (d) $10\tan(0.1x-5)-2x+c$

(e) $2\ln(2x+3)+2e^{-\frac{1}{2}x+2}+c$ (f) $-\frac{2}{2x+3}-\frac{1}{2}e^{2x-\frac{1}{2}}+c$ (g) $x+\ln(x+1)-4\ln(x+2)+c$

(h) $2x-\ln(x+2)+\frac{1}{2}\ln(2x+1)+c$ (i) $-\frac{1}{2x+1}+\ln(2x+1)+c$ **7.** (a) $f(x)=\frac{1}{6}(4x+5)^{3/2}$

(b) $f(x)=2\ln(4x-3)+2$ (c) $f(x)=\frac{1}{2}\sin(2x+3)+1$ (d) $f(x)=2x+\frac{1}{2}e^{-2x+1}+\frac{1}{2}e$

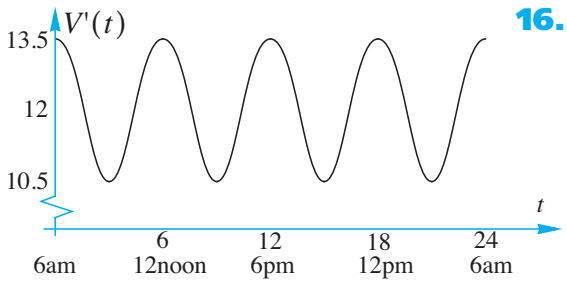
8.14334 **9.**13.19m/sec or 1.19m/sec **10.**2.66cm **11.** $2e^{x/2}-\frac{1}{2}\sin(2x)-2$

12. (a) $p=\frac{a}{a^2+b^2}$, $q=-\frac{b}{a^2+b^2}$ (b) $\frac{1}{13}e^{2x}(2\sin 3x-3\cos 3x)+c$ **13.** (a) $0.25a$

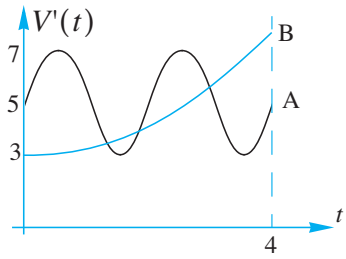
(b) $a\times\left(\frac{1}{2}\right)^{8/3}\approx 0.1575a$ **14.** (b) 666 gm

15. (a) (b) 73.23% (c) ~25.24 litres **16.** (a) (b) 7000 (c) 1.16 day (d) 2 days

15. (a)



16. (a)



EXERCISES 22.4

1. (a) $\frac{15}{2}$ (b) $\frac{38}{3}$ (c) $\frac{5}{36}$ (d) -8 **2.** (a) $\frac{35}{24}$ (b) $\frac{8}{5}\sqrt{2}-2$ (c) -2 (d) 0 (e) $\frac{1}{20}$ (f) $-\frac{4}{3}$ (g) $\frac{7}{6}$

(h) $\frac{5}{6}$ (i) $\frac{20}{3}$ (j) 0 (k) $\frac{20}{3}$ (l) $-\frac{\sqrt{2}}{3}$ **4.** (a) e (b) $2(e^{-2}-e^{-4})$ (c) 0 (d) $2(e-e^{-1})$

(e) e^2+4-e^{-2} (f) $\frac{1}{2}(e-e^5)$ (g) $2\sqrt{e}-3$ (h) $\frac{1}{4}(16e^{1/4}-e^4-15)$ (i) $\frac{1}{2}(e^{-1}-e^3)$

6. (a) $3\ln 2$ (b) $2\ln 5$ (c) $4+4\ln 3$ (d) $\frac{1717}{4}$ (e) $\frac{3}{2}\ln 3$ (f) $2\ln 2$ (g) $\frac{3}{4}$ (h) $4\ln 2-2$ (i) $\ln 2$

8. (a) 1 (b) $\frac{3\sqrt{3}}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) -2 (e) $\frac{\pi^2}{32}-1$ (f) 0 (g) 0 (h) $\frac{\sqrt{3}}{2}-\frac{1}{2}$ (i) 0 **9.** (a) $\frac{31}{5}$

(b) $\frac{7\sqrt{7}}{3}-\sqrt{3}$ (c) 0 (d) $\frac{5}{72}$ (e) $3\sqrt[3]{2}-\frac{3}{2}$ (f) $1-\ln 2$

10. $\ln\left(\frac{21}{5}\right)$ **11.** $\sin 2x+2x\cos 2x$; 0 **12.** (a) $2m-n$ (b) $m+a-b$ (c) $-3n$ (d) $m(2a-b)$

(e) na^2 **13.** (a) $e^{0.1x}+0.1xe^{0.1x}$; $10xe^{0.1x}-100e^{0.1x}+c$ (b) i. 99 accidents

ii. $N = 12t+10te^{0.1t}-100e^{0.1t}+978$ **14.** (a) 1612 subscribers (b) 46220 **15.** (b) ~524 flies

EXERCISES 22.5

1. (a) 4 sq.units. (b) $\frac{32}{3}$ sq.units. (c) 4 sq.units. (d) 36 sq.units. (e) $\frac{1}{6}$ sq.units. **2.** (a) e sq.units.

(b) $\frac{1}{2}(e^4-2-e^2)$ sq.units. (c) $2(e+e^{-1}-2)$ sq.units. (d) $2(e^2-2-e)$ sq.units.

3. (a) $\ln\left(\frac{5}{4}\right)$ sq.units. (b) $2\ln 5$ sq.units. (c) $3\ln 3$ sq.units. (d) 0.5 sq.units. **4.** (a) 2 sq.units.

(b) $\frac{\pi}{2}$ sq.units. (c) $\frac{3}{8}\pi^2+\sqrt{2}-2$ sq.units. (d) $\sqrt{2}$ sq.units. (e) $4\sqrt{3}$ sq.units. **6.** 12 sq.units.

7. $4\left(\sqrt{3}-\frac{1}{3}\right)$ sq.units. **8.** $\ln 2+1.5$ sq.units. **9.** 2 sq.units. **10.** $\frac{37}{12}$ sq.units. **11.** (a) 0.5 sq.units.

(b) 1 sq.unit. (c) $2(\sqrt{6}-\sqrt{2})$ sq.units. **12.** $\frac{8}{3}$ **13.** $-2\tan 2x$; $\frac{1}{4}\ln 2$ sq.units. **14.** (a) $\frac{9}{2}$ sq.units.

(b) 3 sq.units. **15.** (a)1 sq.unit. (b) 10 sq.units or 2 sq.units **16.** (a) $x\ln x-x+c$

(b) 1 sq.unit. **17.** $\frac{14}{3}$ sq.units **18.** (a) $\frac{7}{6}$ sq.units (b) $\frac{9}{2}$ sq.units **19.** i. $\frac{15}{4}$ sq.units ii. $\frac{45}{4}$ sq.units

20. $\frac{22}{3}$ sq.units. **21.** (b) i. $e^{-1}+e-2$ sq. units ii. 1 sq. unit iii. $2\ln(2)$ sq. units

22. (b) 3.05 sq. units **23.** (a) $2y = 3ax - a^3$ (b) $\frac{1}{15}a^5$ sq. units **24.** (a) $1 - e^{-1}$ sq. units

(b) e^{-1} sq. units (c) $1 - e^{-e^{-1}-1} - e^{-1} \approx 0.10066$ sq. units **25.** $a = 16$

EXERCISES 22.6

1. (a) $x = t^3 + 3t + 10, t \geq 0$ (b) $x = 4\sin t + 3\cos t - 1, t \geq 0$

(c) $x = t^2 - 4e^{-\frac{1}{2}t} + 2t + 4, t \geq 0$ **2.** (a) $x = t^3 - t^2, t \geq 0$ (b) 100 (c) $100\frac{8}{27}$ m

3. (a) $x = -\frac{2}{3}(4+t)^{3/2} + 2t + 8$ (b) 6.92 m **4.** $\frac{125}{6}$ m **5.** $\frac{125}{49}$ sec; 63.8 m **6.** (a) $\frac{\pi}{6}$ sec

(b) $\frac{\pi}{2} - 1$ m **7.** 80.37 m **8.** (a) $s(t) = \frac{160}{\pi}\left[1 - \cos\left(\frac{\pi}{16}t\right)\right], t \geq 0$ (b) 86.94 m (c) –6.33 m

(d) 116.78 m **9.** (a) $v = 4 + k - \frac{k}{t^2}, t > 0$ (b) $k = 2$ (c) 52.2 m **10.** (b) 0.0893 m

EXERCISES 22.7

All values are in cubic units

1. 21π **2.** $\pi\ln 5$ **3.** $\frac{\pi}{2}(e^{10} - e^2)$ **4.** π^2 **5.** $\frac{\pi}{2}$ **6.** $\pi\left(\frac{8}{3} - 2\ln 3\right)$ **9.** $\frac{\pi}{5}(5 - 5\sin 1)$

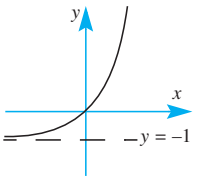
10. $\frac{251}{30}\pi$ **11.** $\frac{242}{5}\pi$ **12.** $\frac{\pi}{4}$ **13.** $\frac{88}{5}\sqrt{3}\pi$ **14.** $\frac{3\pi}{4}$ **15.** $k = 1$

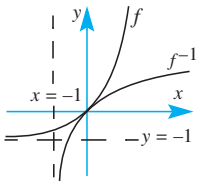
16. $4\pi^2a^2$ **17.** $k = \frac{\pi}{2}$ **18.** $\frac{8\pi}{15}\sqrt{\frac{a}{1+a^2}}\left(\frac{3a^2+2}{1+a^2}\right)$ **19.** (a) two possible solutions: solving

$a^3 - 6a^2 - 36a + 204 = 0$, $a = 4.95331$; solving $a^3 - 6a^2 - 36a - 28 = 0$, then $a = -0.95331$.

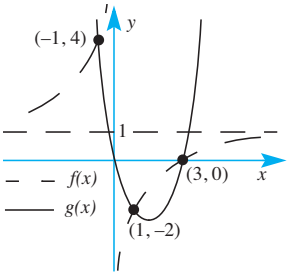
20. $\frac{28}{15}\pi$ **21.** 64π

REVISION EXERCISES – SET A

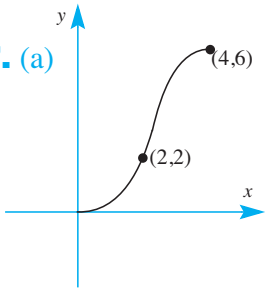
1. –84 **2.** (a) 

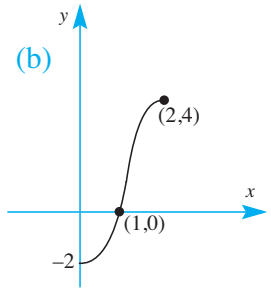
(b) i. $]-1,\infty[$ ii. $f^{-1}(x) = \ln(x+1)$ (c) 

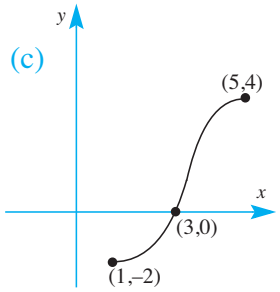
3. 840 **4.** (a) i. 0 ii. 2 (b) $-2 \le x \le 2$ (c) $x \ge 0$ **5.** (a) (1, –2), (–1, 4) and (3, 0)

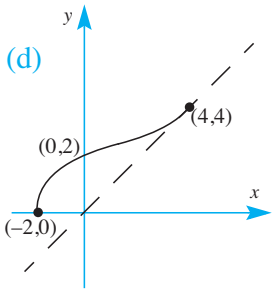
(b) 

6. (a) 2 (b) S = $[0,\infty[$, range = $[1,\infty[$
(c) $f^{-1} : [1,\infty[\mapsto \mathbb{R}$, $f^{-1}(x) = (\ln x)^2$

7. (a) 

(b) 

(c) 

(d) 

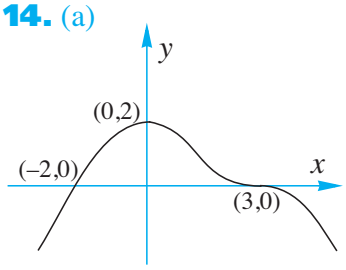
8. (a) i. 512 ii. 2 (b) i. $3x^2h + 3xh^2 + h^3$ ii. $3x^2 + 3xh + h^2$

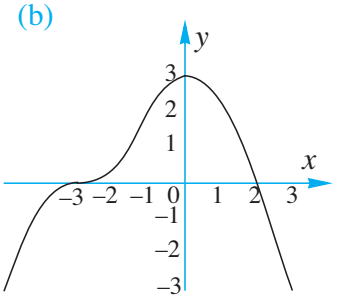
9. (a) i. –1 or 6 ii. $\dfrac{3}{e-1}$ (b) i. $\mathbb{R}\backslash\{3\}$ ii. 0.2 iii. 0

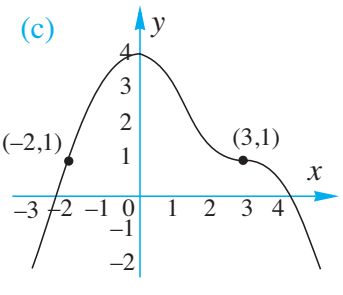
10. (a) i. 2 or 6 ii. $\dfrac{1}{3}(e^2-4)$ (b) i. $0 < x < 1$ ii. $\mathbb{R}$ iii. $\log_e 4 \approx 0.72$ iv. $\dfrac{e^{0.8}}{1+e^{0.8}} \approx 0.69$

11. (a) $g(f(x)) = -\dfrac{2x}{1-x}$, $x \in \mathbb{R}\backslash\{\pm 1\}$ (b) P ≡ (2, 4)

12. (a) i. $x = \dfrac{\ln 6}{\ln 3}$ ii. $\dfrac{15}{7}$ (b) $1+\sqrt{3}$ (c) i. $f(g(x)) = \sqrt{\dfrac{1}{x^2}-1}$, $g(f(x)) = \dfrac{1}{x-1}$ ii. $[-1,1]\backslash\{0\}$

13. 0.5 **14.** (a) 

(b) 

(c) 

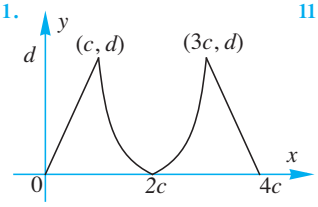
15. (a) $k = 0$ or 1 (b) $0 < x < 3$ **16.** (a) $0 < x < 5$ (b) 70 **17** i. 9 ii. –4 **18.** ±3

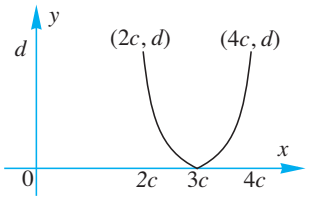
19. (a) $y = -2x$ (b) $\dfrac{x-y}{x+y}$ **20.** (b) $x = -\dfrac{4}{9}, y = \dfrac{1}{9}$ **21.** (b) ii. $p^5 = 3+5p$, $p^4 = 2+3p$

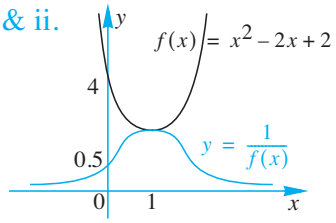
22. (a) $\dfrac{2}{9}$ (b) 59136 **23.** $a = -\dfrac{3}{5}, b = -\dfrac{648}{25}, n = 10$ **24.** (a) ii. $\{\pm 1\}$ (b) i. $y = \sqrt{6(x-3)}$

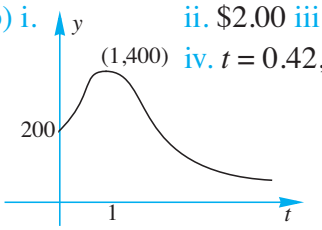
ii. $x = 9, y = 6$ **25.** $1792x^5$ **26.** (a) $\dfrac{5}{2}, -\dfrac{3}{2}$ (b) $\dfrac{3}{2}, -\dfrac{1}{2}$ (c) $\dfrac{17}{2}$ **27.** (a) $\mathbb{R}$ (b) $]-\infty,4]$ (c) $]-\infty,4[$

28. (b) $\dfrac{5}{8}$ sq. units

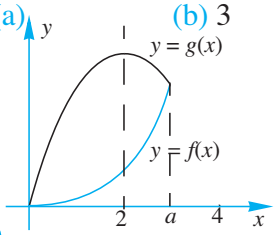
29. i. 

ii. 

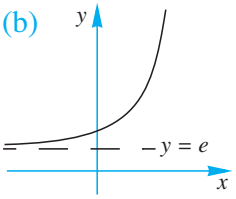
30. (a) i. & ii. 

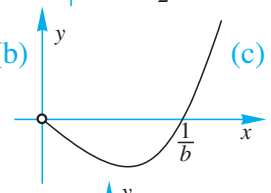
(b) i. 

ii. \$2.00 iii. \$4 iv. $t = 0.42, 1.57$

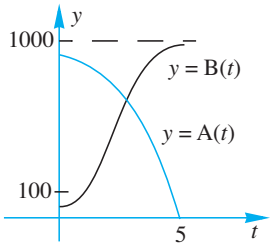
31. (a) 

(b) 3

32. (a) $g^{-1}(x) = e^{2x} + e, x \in \mathbb{R}$
(b) 

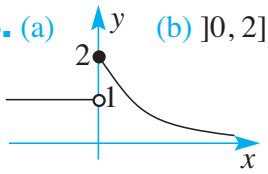
33. (a) i. $]0, \infty[$ ii. $\left]\left(-\frac{a}{eb}\right), \infty\right[$ (b) 

(c) $(1, a\log_e b)$ (d) $x = b^{\frac{1}{x}-1}$

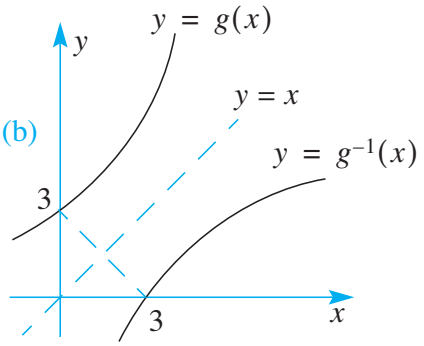
34. (a) $a = -36, b = 900$
(b) (c) 20 (d) 1000 (e) $t > 1.95$ (f) 

t	1	2	3.5	5
$B(t)$	131.04	527.02	957.23	997.78

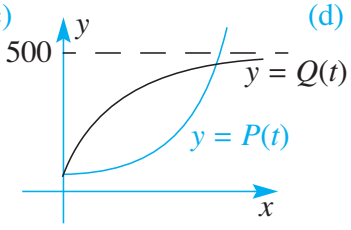
35. (a) 150 cm (b) 138 cm (c) 94 hrs (d) $[0, 94]$ (e) $h^{-1}(x) = \dfrac{12.5 - \sqrt{t}}{0.13}$ (f) Use g.c (g) 17.3 hrs

36. (a) 

(b) $]0, 2]$ (c) No ($x = 0$) **37.** 78 **38.** (a) 0 (b) $-\sqrt{2}$ (c) $r_f \not\subseteq d_g$, doesn't exist

39. $-\frac{63}{8}x^5$ **40.** (a) $g^{-1}(x) = -1 + \sqrt{x-2}, x \geq 2$ (b) 

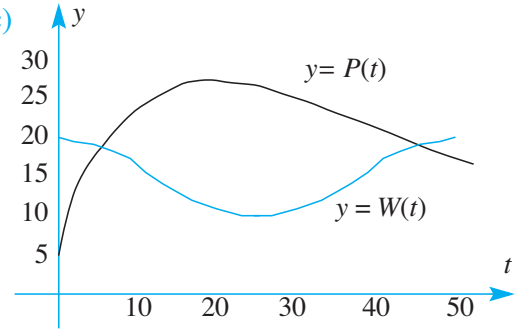
(c) no

41. ii. $h(x) = 4 - x, x \geq 0$, range $=]-\infty, 4]$ **42.** i. use g.c ii. $f^{-1}(x) = -\log_e(1-x), x < 1$
iii. use g.c **43.** –10 **44.** (a) $r_g \subseteq d_f \Rightarrow fog$ exists; $r_f \not\subseteq d_g \Rightarrow gof$ doesn't exist
(b) $x < -2$ or $x > 2$ (c) $\mathbf{S} =]-3, 2[$ **45.** (a) $f^{-1}(x) = (2-x)^2, x < 2$ (b) $r_g \not\subseteq d_{f^{-1}} \Rightarrow f^{-1}og$ does not exist; $r_{f^{-1}} \subseteq d_g \Rightarrow gof^{-1}$ exists (c) $F(x) = x - 2, x \leq 2$ **46.** (a) i. 50 ii. $50e \approx 135.9$
(c) 

(d) i. 50 ii. 334.5 (f) Increasing at a decreasing rate
(g) ~ 460 wasps
(h) ii. $t = 0$ and $t = 10\log_e 9$

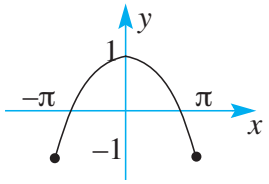
REVISION EXERCISES – SET B

1. (a) 189 (b) 99 (c) –96 (d) 36 **2.** (b) –65 **3.** (b) 23.9 km (c) 26.4° (d) 15.3 km (e) 107.5
4. (a) i. A: \$49000; B: \$52400; C: \$19200 ii. A: \$502400; B: \$506100; C: \$379400 (b) 4.6%
(c) i. 14 months ii. C never reaches its target **5.** (a) $r = 0.5$ (b) 62.5 cm **6.** (b) 26°34′ or 135°
7. (b) $\frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}$ **8.** 28 **9.** (a) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ (b) $0, \frac{\pi}{2}, \pi, 2\pi$ **10.** (a) max value is $\frac{17}{2}$ for $x = \frac{\pi}{2} + 2k\pi$ or $x = \frac{3\pi}{2} + 2k\pi$, where k is an integer; min value is $\frac{17}{5}$ for $x = k\pi$, where k is an integer (b) $\frac{\pi}{3}, \frac{5\pi}{3}$ **11.** (a) $u_n = 74 - 6n$ (b) $n = \frac{1}{6}(74 - p)$ (c) $\frac{1}{12}(74 - p)(68 + p)$, 420
12. $\frac{24(4\sqrt{3} - 3)}{39}$ **14.** (a) 60°, 109°28′, 250°32′, 300° (b) i. $2\text{cosec}\theta$ ii. $\frac{\pi}{3}, \frac{2\pi}{3}$ **15.** (a) ~342
(b) 20 terms (c) $0 < x < 2$ (d) $\{1, 3, 8, 18, \ldots\}$ (e) $u_n = 23 - 3n$ (f) \$4131.45 **16.** (a) $-\frac{1}{2}$
(b) 4 **17.** (a) 120° (b) $14\sqrt{3}$ cm² **18.** (a) i. $0.3\sqrt{3}$ m ii. $0.2\sqrt{3}$ m (b) ~1.15 m (c) 73°13′
19. (a) $\frac{\pi}{3}, \frac{4\pi}{3}$ (b) $\left\{x \middle| \frac{\pi}{3} < x < \frac{4\pi}{3}\right\}$ **20.** (a) 8 cm (b) 28°4′ **21.** 3 **22.** (a) $\left\{\frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}\right\}$
(b) $\frac{\pi}{3}$ **23.** (a) \$77156.10 (b) $u_1 = -\sqrt{3}, u_3 = -3\sqrt{3}$ **24.** (a) $f(x) = 3\cos(2x)$ (b) $\left\{\frac{7\pi}{6}\right\}$
(c) 3 **25.** (b) i. BP = 660 m, PQ = 688 m **26.** 216° **27.** (b) 906 m **28.** (a) 38°40′
(b) 0.08004 m² (c) \$493.71 **29.** (a) $\tan\alpha = -\frac{1 + \sqrt{5}}{2}$ (b) range = [3, 3.5] (c) i. 3 ii. 2
30. (a) i. $W(4) = 19.38, P(4) = 14.82$ ii. $W(20) = 10.95, P(20) = 27.02$
iii. $W(35) = 13.45, P(35) = 23.25$ (b) Amp = 5, period = 50 weeks
(c)



(d) \$27.07 (e) during 7th & 46th week
31. (a) \$49000, \$47900, \$46690 (b) \$34062.58
(c) 18.8 yrs (d) ~\$248564
32. (a) ii. 26 cards (b) 26, 40, 57, 77
(c) $a = 3, b = -1$ (d) 155 cards
(e) $t_n = \frac{n}{2}(3n + 1)$
33. (a) ~2.77 m (b) i. 3.0 m ii. 2.0 m (c) 4.15 pm
(d) use g.c. (e) $2\frac{1}{6} < t < 6\frac{1}{3}$

34. 1.262 ha 35. $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$ **36.** (a) $x = -\frac{2\pi}{3}, \frac{2\pi}{3}$ (b)



(c) $-\frac{2\pi}{3} < x < \frac{2\pi}{3}$

37. 1623 m **38.** (a) 19.5°C (b) $D(t) = -1 + 2\cos\left(\frac{\pi}{12}t\right)$ (d) use g.c (e) 8 am to midnight
39. 1939 m **40.** (a) ii. $N_0 = 2000, \alpha = 10$ (b) 2000, 2200, 2420, 2662, 2988.2 (c) 52 hrs
(d) 176995

41. (a) $(4-\pi)$ cm^2 (b) $\frac{(4-\pi)}{2}$ cm^2 (c) ii. $r = \frac{1}{2}$ iii. $A_n = (4-\pi)\times\left(\frac{1}{2}\right)^{n-1}, n = 1, 2\ldots$

(d) i. $\frac{31}{16}(4-\pi)$ cm^2 ii. $2(4-\pi)$ cm^2 (e) geometric

REVISION EXERCISES – SET C

1. (a) $-7\boldsymbol{i}+6\boldsymbol{j}+\boldsymbol{k}$ (b) -8 (c) $a = \frac{1}{\sqrt{3}}(\boldsymbol{i}+\boldsymbol{j}+\boldsymbol{k})$ **2.** i. $x = \frac{7}{2}+\lambda, y = \lambda, z = \frac{9}{2}+5\lambda$

ii. $\frac{x-3.5}{1} = \frac{y}{1} = \frac{z-4.5}{5}$ **3.** 0 **4.** $k = 1$ or 3 **5.** $t = 2, (16, -8, 4)$ **6.** $A^2 = \mathrm{I}_{3\times 3}$; $(-1, -3, 4)$

7. $BX = \frac{1}{2}A$, $X = \frac{1}{18}BA$, $AX = \frac{1}{18}ABA$ **8.** (a) $3\boldsymbol{i}-\boldsymbol{j}-2\boldsymbol{k}$ (b) 100° (c) $4\boldsymbol{i}-3\boldsymbol{j}-3\boldsymbol{k}$

9. (b) $A^{-1} = \frac{1}{3}(A-2\mathrm{I})$ **10.** (a) $|\boldsymbol{r}_B|_{min} = 2\sqrt{2}$ (b) $t = 5, b = \frac{2}{5}$ **11.** (a) $\frac{1}{a-2}\begin{bmatrix}-3\\2a-1\end{bmatrix}$

(b) $a = 2$ (c) $x = -3, y = 5$ **12.** (a) i. 90° ii. $\frac{7}{2}\sqrt{26}$ unit^2 (b) i. $\boldsymbol{s}+3\boldsymbol{p}$ ii. $\boldsymbol{s}+2\boldsymbol{p}$ iii. $\frac{1}{2}\boldsymbol{s}+2\boldsymbol{p}$

iv. $-\frac{1}{2}\boldsymbol{s}+2\boldsymbol{p}$ **13.** (a) 1 (b) $A^n = \begin{bmatrix}2^n & (2^n-1)a\\0 & 1\end{bmatrix}$ (c) $a = -\frac{2}{9}$ **14.** (a) 27° (b) $\frac{1}{2}\sqrt{17}$ unit^2

15. (a) i. $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}2\\-3\end{pmatrix}+\lambda\begin{pmatrix}3\\7\end{pmatrix}, \lambda\in\mathbb{R}$ ii. $x = 2+3\lambda, y = -3+7\lambda, \lambda\in\mathbb{R}$ iii. $\frac{x-2}{3} = \frac{y+3}{7}$

(b) $-\boldsymbol{i}+11\boldsymbol{j}$ (c) i. no ii. lines are skew **16.** 28°35′ **17.** (a) $a = \frac{3}{2}$ (b) $b = \frac{3}{2}, c = \frac{1}{3}$

18. $\frac{4}{\sqrt{77}}\left(-\frac{5}{4}\boldsymbol{i}+\boldsymbol{j}+\frac{3}{2}\boldsymbol{k}\right)$ or $-\frac{4}{\sqrt{77}}\left(-\frac{5}{4}\boldsymbol{i}+\boldsymbol{j}+\frac{3}{2}\boldsymbol{k}\right)$ **19.** $k = -1, 1$ or 2 **20.** (a) i. $\begin{bmatrix}2\\5\end{bmatrix}$ ii. $\frac{1}{2}\begin{bmatrix}1&0\\2&2\end{bmatrix}$

(b) $x+y-z = 1, 3x+4y+6z = 7$ (c) $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}-3\\4\\0\end{pmatrix}+\lambda\begin{pmatrix}10\\-9\\1\end{pmatrix}$ **21.** (a) 5 (b) $\frac{5}{3}\sqrt{5}$

22. $\boldsymbol{OA} = 2\boldsymbol{i}-2\boldsymbol{j}+\boldsymbol{k}$, $\boldsymbol{OB} = 4\boldsymbol{i}-3\boldsymbol{k}$; 70°32′ **23.** $AB = \mathrm{I}$; $x = 3, y = 0, z = -1$

24. (a) [11] (b) $\begin{bmatrix}3&6\\4&8\end{bmatrix}$ (c) $\begin{bmatrix}2&4\\5&8\end{bmatrix}$ (d) does not exist **25.** Yes **26.** (a) i. $(1, -1, 2)$; $\left(\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)$

ii. $(3, 6, 2)$; $\left(\frac{3}{7}, \frac{6}{7}, \frac{2}{7}\right)$ (b) lines do not meet **27.** $\frac{x-1}{3} = \frac{y-2}{2} = \frac{z+3}{1}$ **28.** $\left(\frac{2}{5}, \frac{23}{5}\right)$

29. $\frac{2}{3}$ or 2 **30.** (a) $\boldsymbol{r}_A = \begin{pmatrix}0\\80000\end{pmatrix}+t\begin{pmatrix}3\\-2\end{pmatrix}$ (b) $\begin{pmatrix}21600\\65600\end{pmatrix}$ (units in metres) (c) They do not

collide **31.** (a) $\boldsymbol{r} = \begin{pmatrix}4\\2\end{pmatrix}+t\begin{pmatrix}2\\3\end{pmatrix}$ (b) $\boldsymbol{LP} = \begin{pmatrix}-21\\-11\end{pmatrix}+t\begin{pmatrix}2\\3\end{pmatrix}$ (c) ~ 129.31 km

REVISION EXERCISES – SET D

1. (a) $\frac{1}{4}$ (b) $\frac{3}{8}$ (c) 0.3169 **2.** 0.0228 **3.** (a) 0.12 (b) 0.6087 **4.** (a) 0.89 (b) $\frac{21}{40}$ (c) $\frac{40}{89}$

5. (a) 0.46 (b) $\frac{9}{23}$ **6.** (a) 3326400 (b) i. $\frac{2}{11}$ ii. $\frac{2}{77}$ **7.** (a) 0.9772 (b) 0.3413 **8.** (a) 0.936 (b) 5

9. (a) 792 (b) 35 **10.** (a) 151200 (b) 0.1512 **11.** 0.2852 **12.** $\frac{128}{850} \approx 0.1506$ **13.** (a) 0.10 (b) 0.40

(c) $(x, P(X = x))$ values are: $(0, 0.40), (1, 0.50), (2, 0.10)$ (d) $E(X) = 0.70, var(X) = 0.41$
14. (a) 0.8664 (b) 0.7210 (c) 0.9034 (d) $9.8855 < Y < 10.2145$ (e) 79.3350

15. (a) 315 (b) 17280 **16.** $\frac{193}{512}$ **17.** (a) $\frac{2}{3}$ (b) $\frac{1}{2}$ **18.** (a) $P(X = x) = \frac{1}{6} \times \left(\frac{5}{6}\right)^x, x = 0, 1, \ldots$

(i.e., Geometric) (b) i. 0.0670 ii. 0.4019 iii. $\frac{1}{6}$ **19.** (a) $\frac{13}{44}$ (b) $\frac{9}{44}$

20. (b) $(x, P(X = x))$ values are: $\left(1, \frac{9}{25}\right), \left(3, \frac{7}{25}\right), \left(5, \frac{5}{25}\right), \left(10, \frac{3}{25}\right), \left(20, \frac{1}{25}\right)$

(c) $E(X) = \frac{105}{25} \approx 4.2$, $\text{var}(X) = \frac{11400}{625} \approx 18.24$ (d) 0.00064 **21.** (a) 0.3085 (b) 0.0091 (c) 0.1587

22. 100 **23.** (a) $\frac{1}{2}$ (b) $\frac{1}{7}$ (c) $\frac{2}{7}$ **24.** (b) $(x, P(X = x))$ values are: (1,0.4), (2,0.3), (3,0.2),(4,0.1)
(c) i. 2 ii. 5 iii. 3 **25.** i. 0.8186 ii. 0.1585

26. (a) $(x, P(X = x))$ values are: $\left(0, \frac{3}{16}\right), \left(1, \frac{7}{16}\right), \left(2, \frac{5}{16}\right), \left(3, \frac{1}{16}\right)$ (b) ii. 0.0064 iii. 0.7705

27. $\mu = 0.9586, \sigma = 0.0252$ **28.** (a) $\frac{10}{21}$ (b) i. 0.3085 ii. 0.1747 **29.** (a) i. 0.8 ii. 0.25

(b) i. 0.4 ii. $E(X) = 0.8, var(X) = \frac{14}{25}$ **30.** (a) i. $\frac{1}{8}$ ii. $\frac{47}{72}$ iii. $\frac{1}{8}$ iv. $\frac{47}{72}$ v. $\frac{9}{47}$ **31.** $\frac{189}{8192}$

32. $\frac{43}{60} \approx 0.7167$ **33.** $\frac{117}{145} \approx 0.8069$ **34.** i. $(x, P(X = x))$ values are: $\left(0, \frac{1}{6}\right), \left(1, \frac{1}{3}\right), \left(2, \frac{1}{2}\right)$;

$E(X) = \frac{4}{3}, var(X) = \frac{5}{9}$ ii. $\frac{2}{3}$ iii. $\frac{5}{24}$ **35.** i. 0.4 ii. 0.096 iii. 0.225 iv. 0.635 **36.** (a) $\frac{3}{5}$

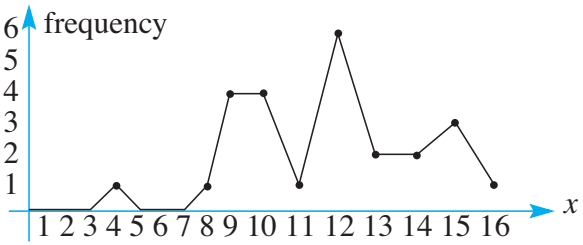
(b) $(x, P(X = x))$ values are: $\left(0, \frac{4}{25}\right), \left(1, \frac{12}{25}\right), \left(2, \frac{9}{25}\right)$ (c) $E(X) = 1.2, var(X) = 0.48$

(d) $\frac{3}{7}$ **37.** (a) i. $\frac{8}{15}$ ii. $\frac{7}{15}$ iii. $\frac{1}{5}$ iv. $\frac{4}{5}$ v. $\frac{4}{7}$ (b) $\frac{x(p-q)+100q}{100}$ **38.** $\frac{2}{3}$ **39.** (a) 0.1359

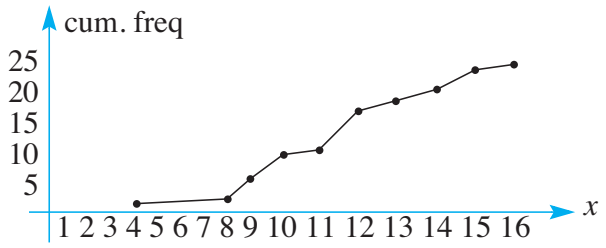
(b) 137.22 (c) $137\frac{1}{3}$ (d) $a = 141.21$ **40.** i. $\frac{2}{3}$ ii. $\frac{2}{9}$ iii. not independent

41. i. $b + 6a$ ii. $0 \le b \le \frac{1}{3}$ **42.** i. 0.081 ii. $\frac{4}{13}$ **43.** (a) 0.0169 (b) i. 0.9342 ii. 127 iii. 0.008

44. i. 0.1587 ii. 0.7745 iii. \$0.23 **45.** (a)



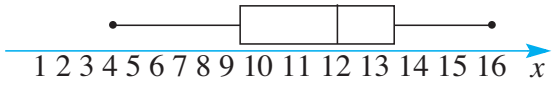
ii.



(b) i. 11.44 ii. 2.6695

(c) i. $x_{min} = 4$, $x_{max} = 16$, med = 12,

$Q_3 = 13.5$, $Q_1 = 9.5$



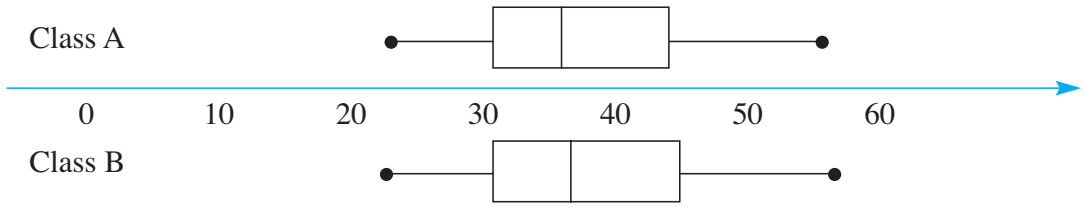
ii. med = 12, mode = 12 iii. 4

46. (a) i. 0.24 ii. 0.36 (b) $172 + 0.96Q$ (c) $Q > 29.17$

47. (a) use g.c (b) i. $\bar{x}_A = 37.35$, $\bar{x}_B = 37.31$ ii. A: $s_n = 8.801$, B: $s_n = 9.025$

(c) i. Class A: $x_{min} = 23$, $x_{max} = 56$, med = 36, $Q_3 = 43$, $Q_1 = 31$

Class B: $x_{min} = 22$, $x_{max} = 57$, med = 37, $Q_3 = 44$, $Q_1 = 31$



ii. Class A: med = 36; multimodal – 34, 35, 39, 43, 48

Class B: med = 37; multimodal – 27, 34, 38, 42, 49

iii. Class A: IQR = 12, Class B: IQR = 13 (d) Results from both classes are very close, however, Class B does slightly better as it has a larger median as well as the larger maximum value.

REVISION EXERCISES – SET E

1. (a) $\frac{x}{\sqrt{x^2+4}}$ (b) $2\cos 2x - 2(2x-1)\sin 2x$ **2.** $30\frac{1}{3} + \log_e 4$ **3.** (a) $\frac{\pi}{3}, \frac{4\pi}{3}$

(b) $\left\{x \mid \frac{\pi}{3} < x < \frac{4\pi}{3}\right\}$ (c) 4 sq. units **4.** (a) 19.8°C (b) 1.6°C per minute (c) 17.3 min

5. (a) $x \in [-1, 0[\cup]0, \infty[$ (b) $x \in]-\infty, 0[\cup [2, \infty[$ **6.** 10 m **7.** (a) $\frac{4x}{(x^2+1)^2}$

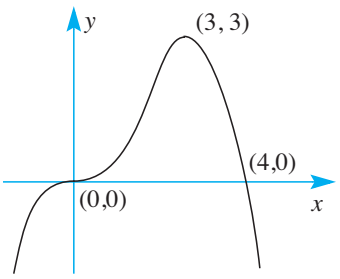
(b) $-4\sin 2x\cos 2x$ (or $-2\sin 4x$) **8.** (a) i. 0 ii. 2 (b) $x \in [-2, 2]$ (c) $x \geq 0$

(d) $-\frac{x}{\sqrt{4-x^2}}, -2 < x < 2$ **9.** (a) $-\frac{2+h}{(1+h)^2}, h \neq 0$ (b) –2 **10.** (a) 74 (b) 0.69 **11.** 1.455 ms^{-1}

12. (a) Absolute maximum at $\left(\pm\frac{1}{\sqrt{2}}, 1\right)$; local min at (0, 0); x-intercept at (±1, 0)

(b) Local min at $\left(\pm\frac{1}{\sqrt{2}}, 1\right)$; asymptotes at $x = \pm 1$, $y = 0$. **13.** i. $6\cos 2x\sin^2 2x$ ii. $\frac{x+3}{(2x+3)^{3/2}}$

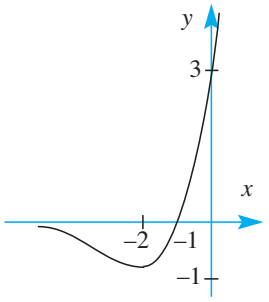
14. $\frac{1}{2}(e^{2x} - 4x + e^{-2x}) + c$ **15.** 720 m^3 **16.** (a)



(b) 98°

17. (b) i. 2 ii. 72 cm^3 **18.** (a) Area = $A = \frac{8}{15}h^{3/2}$, Volume = $V = 0.48h^{3/2}$ (b) $\frac{5}{144}$ m/min

19. (a) $(-1,4),(1,-2),(3,0)$ (b) use g.c (c) $\frac{16}{3} - 3\log_e 3$ sq. units **20.**



21. (a) $h = \frac{1000}{\pi r^2}$ (b) radius = 5 cm, height = 12.7 cm **22.** i. $-\frac{3x}{\sqrt{1-3x^2}}$ ii. $\frac{e^x}{(1+e^x)^2}$

23. (a) $3x^2h + 3xh^2 + h^3$ (b) $3x^2 + 3xh + h^2$ (c) $3x^2$ **24.** (a) $2 - \frac{3}{4}\sqrt{3}$ (b) $\log_e 3$

25. (a) $p'(t) = 0.8(1-0.02t)e^{-0.02t}$ (b) ~ 38.3 million (c) i. decreasing ii. `0.1 million/year (d) 50 years time, i.e., 2030; 42.2 million **26.** 76222 cm^3

27. (a) $A \equiv (-1,5), B \equiv (1,3), C \equiv (4,0)$ (b) use g.c (c) $12 - 4\log_e 4$ sq. units

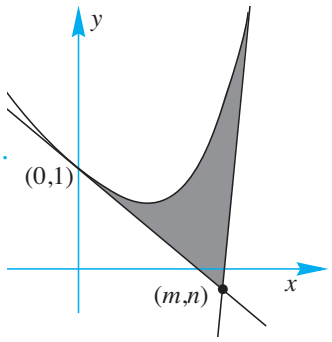
28. i. $\frac{1+\cos x + x\sin x}{(1+\cos x)^2}$ ii. $\frac{x}{x^2+1}$ **29.** (a) $12 + 6h + h^2, h \neq 0$ (b) 12 **30.** (a) $A \equiv (2, 2e^{-1})$

(b) i. $y = x$ ii. $\frac{d}{dx}(xe^{-x/k}) = \left(1 - \frac{x}{2}\right)e^{-x/2}$ iii. $4 - 2(2+a)e^{-a/2} - \frac{1}{2}a^2$ (c) i. $(2x - x^2)e^{-x}$

ii. $\pi(2 - 10e^{-2})$ cubic units **31.** (a) $\log_e x$ (b) $2(\log_e 2 - 1)$ **32.** (a) i. 283 sec ii. 250 sec

(c) 244 sec **33.** (a) $A \equiv \left(\frac{1}{2}ln2, 2(1-ln2)\right)$ (b) $\frac{1}{2}(e^2 - 5)$ sq. units (c) i. At (0,1): $y = -2x + 1$

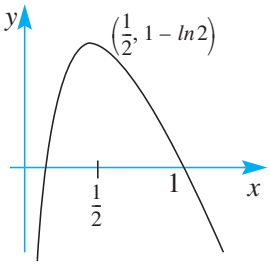
At $(1, e^2-4)$: $y = (2e^2-4)x - e^2$ ii.



iii. $\frac{1}{2}(e^2 - 5)$ sq. units

(d) ii. $\frac{\pi}{12}(3e^4 - 24e^2 + 37)$ cubic units **34.** (b) i. $0 < x < 0.5$ ii. $x = 0.5$ iii. $x < 0$ or $x > 0.5$

(c) $\left(\frac{1}{2}, 1 - ln2\right)$ (d)



(e) i. $y = -x + 1$ ii. $y = x - 1$ (f) i. $\frac{3}{8} - \frac{1}{2}ln2$ sq. units

ii. $\frac{1}{8} + \frac{1}{2}ln2$ sq. units **35.** (a) 4.20 (b) i. $\frac{1}{2}\cos\frac{1}{4}t$ ii. –0.40 **36.** $\pi\log_e 3$ cubic units

37. $a = -1, b = 6, c = -9$ **38.** (a) $\frac{4\sqrt{3}}{3}$ (b) i. $-6\sin 3x\cos 3x$ ii. $\frac{x}{2} + \frac{1}{12}\sin 6x + c$

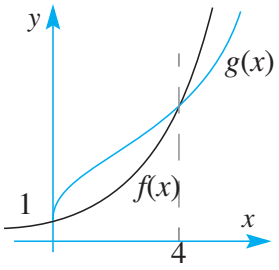
39. $A = 0, B = 0.5$ **40.** (a) $\frac{7}{12}$ sq. units (b) $\frac{7}{15}\pi$ cubic units **41.** (a) $V = \pi r^2 h + \frac{4}{3}\pi r^3$

(b) $P = 2\pi krh + 6\pi kr^2$ (c) $P = \frac{2kV}{r} + \frac{10\pi}{3}kr^2$ (d) $0 < r < \left(\frac{3V}{4\pi}\right)^{1/3}$ (e) $r = \left(\frac{3V}{10\pi}\right)^{1/3}$

42. (b) $\left[\frac{a}{4}, \frac{a}{2}\right]$ (c) $\frac{\sqrt{3}}{36}a^3$ cubic units **43.** (a) $2x\log_e x + x$; $2\log_e 2 - \frac{3}{4}$ (c) $(1, e)$; $y = ex$

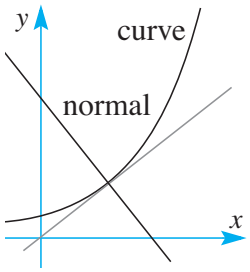
(d) $\left(\frac{1}{2}e - 1\right)$ sq. units **44.** $x_1 = \frac{\pi}{6}, x_2 = \frac{\pi}{4}, x_3 = \frac{\pi}{3}$ **45.** (a) $a = 2$; $f^{-1}(x) = 2 + \sqrt{x}$, $x \geq 0$

(b) $\frac{136}{3}\pi$ cubic units **46.** (a) i. $\mathbb{R}$ ii. $[0, \infty[$ (b) 0.5 (c) $\{0, 4\}$ (d) i.



ii. $\{x \mid x > 4\}$ (e) i. 4 sq. units ii. $\frac{\pi}{2}(e^4 + 3)$ cubic units **48.** i. $-6\sin 2x\cos^2 2x$ ii. $\frac{1 - 2x^2}{\sqrt{1 - x^2}}$

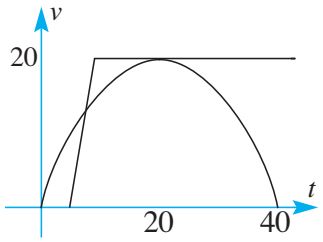
49. (a) $\sin x + x\cos x$ (b) $x\sin x + \cos x$ **50.** (a) $y = -ex + e + e^{-1}$ (b)



(c) $\frac{1}{2}e + e^{-2}$ sq. units **51.** (a) $\frac{4}{3}$ sq. units (b) $\frac{64}{15}\pi$ cubic units **52.** (a) $[0, 5]$ (b) use g.c (c) 0.625

(d) $a = \frac{1}{2} - \frac{1}{5}t, 0 \leq t \leq 5$ **53.** (c) Minimum, $3\pi a^2\left(\frac{5}{3}\right)^{1/3}$; Maximum $3\pi a^2\left(\frac{9}{4}\right)^{1/3}$

54. (a)



(b) 30 seconds (c) $116\frac{2}{3}$ metres **55.** (a) $-e^{-x}(\cos x + \sin x)$

(b) 1 **56.** (a) $-\frac{1}{t^2} + 1$ (b) $\ln t + 2t + \frac{1}{2}t^2 + c$ **57.** (a) use g.c (b) $A(x) = 2x\cos x, 0 < x < \frac{\pi}{2}$

(d) 1.12 sq. units

Note: A Solutions Manual – which includes fully worked solutions to almost every question in the textbook is available. Check the ibid press website for details as well as further updates on answers and solutions.